Analysis of Mechanisms Using Bond Graphs

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Abstract:

This paper demonstrates the use bond graph techniques for modeling of planar mechanisms with a clear insight into system elements. The method eliminates the derivative causalities. Illustration of state space equation for a five bar mechanism is presented.

1. Introduction

Analysis of planar mechanisms is usually done by classical techniques like graphical method, vector approach, dual number approach or using software packages like ADAMS [1]. Many programs have been written and discussed in literature. Unfortunately they are non generic, tedious, cumbersome and error prone [2]. Several programmes are available commercially for use on computers. Although computer solutions are impressive, they are time consuming, expensive and are trial and error solutions [3].

For analysis of mechanisms with more than four links the equations become non-linear. Lagrange and Hamilton methods enable one to derive the differential equations of any given system. Most of these equations are non-linear which can be solved numerically with the help of computers. The job of deriving differential equations has to be performed by human being while job of solving them is most often performed by a computer [4]. In 1959 Paynter of MIT invented a procedure by which task of deriving differential equations can also be performed by a computer to obtain complete understanding of the system behavior. The language in which any system can be represented in graphical form and can be conveyed to computer is called bond graph [5]. Apart from offering advantages in deriving system equations, the representation of a system in terms of bond graphs also brings out the hidden interactions between elements and offers a better understanding of the system dynamics [6].

To create a bond graph for a mechanical system a schematic sketch of the system is made.The following three methods are found effective to create bond graphs for mechanical systems [7]: 1) Method of Flow Map (MFM), 2) Method of Effort Map (MEM) 3) Method of Mixed Map (MM).

The method of flow map is based on the fact that the single port mechanical capacitance C or resistance R element may be attached to a 1-junction where the relative velocity of ends of these elements is available. Single port I elements may be attached to 1-junction, where absolute velocities of generalized inertias are available.

The method of effort map helps to determine the power directions of the bonds attached to 1-junction. It is based on the principal that positive value of force accelerates the inertia point in appositive sense. In bond graph the tensile force in the spring and reactive force on a damper are taken as positive. C and R elements are directly put on 0 - junctions depicting the force in them.

In the method of mixed map the 1-junctions on which the forces accelerating generalized inertia points are added correspond to velocities of these inertial points in MEM. Now if a part of a system is modeled by MEM, then the 1-junction depicting these velocities may be used to create a bond graph for the rest of the system following MFM.

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The method of flow map is proposed here for analysis of five bar mechanism as this can be analyzed as a single port mechanical element. The models of mechanisms are created by using the relation between the input speeds and velocity components of centre of masses of rigid links and their angular velocities. If the links are taken to be rigid, this approach is very convenient as the library LINKPAC may be directly used [7].

2. Bond Graph for Five-Bar Mechanism

This is a two degree freedom mechanism. The schematic diagram shown in figure 1 is representative of typical five bar mechanism. The mechanism is driven by a flexible drive shaft at constant angular speed ω_2.The motion of the mechanism is constrained by constraint force applied on link 3 in the form of a controlled constant angular velocity ω_3 There is an elastic load at the out put link 5. The bond graph of mechanism is shown in figure. 2. [8].

The input crank rotation appears at the $1\dot{\theta}_2$ junction in the graph. The velocity source SF_1 drives the crank through the elastic shaft represented by the 1-C-R structure. Joint resistance is modeled by the R-element of coefficient b_{21}. The controlled motion is applied (restriction) at $1\dot{\theta}_3$ - junction and the velocity source SF_2 controls the motion of link 3 through the elastic shaft represented by 1-C-R. structure. Joint resistance is modeled by the R element of coefficient b_{23}.

The linear motion of the couplar center of mass is resolved into principal direction x_4, y_4 and is determined from the crank rotation through the transformers junction - $1\dot{\theta}_4$ with I element models the rotation of the couplar. The resistance of the joint between the second crank and the couplar is modeled by R-element of coefficient b_{43} at the 0-junction connecting the junction $1\dot{\theta}_3$ and $1\dot{\theta}_4$. Link 5 receives the rotational motion from the crank through link 4. This is represented by the TF connecting $1\dot{\theta}_2$ - junction to $1\dot{\theta}_5$ -junction. Further the joint resistance between link 4 and link 5 is modeled by R-element of coefficient b_{54} at 0-junction joining the angular motion of link 4 and link 5. Moduli of transformer can be obtained by from kinematic analysis of the mechanism or by using the LINKPAC Subroutines [6]. From the bond graph it is seen that all inertia elements except I_1 are with differential causalities indicating that the links connected to the crank have dependent motions. These differential casualties can be eliminated by using pads. The resulting bond graph is shown in figure 3.

3. State Space Formulation of Mechanism Dynamic Equations

If all derivative causalities are eliminated as described by Margolis and Karnopp [1] the state space formulation can be obtained from the bond graph shown in fig. 4(b) using the standard procedure in the following form

$$\dot{X}(t) = A(s,r)\,X(t) + B(s,r)\,U(t) \qquad (1)$$

Where, X (t) is the system state vector consisting of various components of link momentum (p) and displacements (q) and U (t) is the input source vector. The matrices A(s,r) and B(s,r) depend on the system parameters (s) and TF moduli (r) [9].

The free body diagram shown in figure 4 (a) helps to determine the five connecting forces. The figure 4(b) shows corresponding bond graph in which artificial compliances C6, C7, C8, and C9 are added such that all inertia elements of link 4 are integrally causalled and are connected to the input degree of freedom through the TFs. This results in a situation where the derivation of system equations become straightforward and does not require the computation of derivatives of TF moduli [10].

The state variables of the mechanism are the momenta of masses P_1, P_2, P_3, P_4, P_5 (link momenta) and position of springs or compliance elements q_6, q_7, q_8, q_9 (link displacements).

The system equations are generated by answering the following two questions

1. What do all the elements give to the system expressed in terms of system variables?

2. What does the system give to storage elements with integral causality?

The inertial elements I_1, I_2, I_3, I_4, and I_5 gives to the flow (f) to the system and receive the effort (e) from the system

$$f_1 = \frac{p_1}{I_1};\ f_2 = \frac{p_2}{I_2};\ f_3 = \frac{p_3}{I_3};\ f_4 = \frac{p_4}{I_4};\ f_5 = \frac{p_5}{I_5} \qquad 2$$

$$e_1 = \dot{p}_1;\ e_2 = \dot{p}_2;\ e_3 = \dot{p}_3;\ e_4 = \dot{p}_4;\ e_5 = \dot{p}_5 \qquad 3$$

The compliance element C_6, C_7, C_8, C_9 gives efforts e_6, e_7, e_8, e_9 to the system. These efforts expressed in terms of state variables are

$$e_6 = k_6\dot{q}_6;\ e_7 = k_7\dot{q}_7;\ e_8 = k_8\dot{q}_8;\ e_9 = k_9\dot{q}_9 \qquad 4$$

The resistances R_{12} and R_{24} gives efforts e_{12} and e_{24} to the system and in terms of state variables

$$e_{12} = Rf_{12} = Rf_1 = R\frac{p_1}{I_1} \text{ (} f_{12} \text{ beomg mot a state variable)} \qquad 5$$

The effort sources SE^s gives the efforts $e_{11}=F_1$ (t) and $e_{10}=F_2$ (t) to the system Now

$$\dot{p}_1 = e_1 = e_{11} - e_{12} - e_{13} \text{ (Substituting respective values)} \qquad 6$$

$$\dot{p}_1 = F_1(t) + r_1F_2(t) - R\left(\frac{p_1}{I_1}\right) - Rr_1^2\left(\frac{p_1}{I_1}\right) - r_1k_6q_2 - r_1r_2k_7q_7 - r_1r_2k_7q_7 - r_1r_3k_8q_8 - r_1r_4k_8q_8 \qquad (7)$$

$$\dot{p}_2 = k_6q_6 \qquad (8)$$

$$\dot{p}_3 = k_7q_7 \qquad (9)$$

$$\dot{p}_4 = k_8q_8 - m_4g \qquad (10)$$

$$\dot{p}_5 = k_9q_9 \qquad (11)$$

$$\dot{q}_6 = f_6 = r_1\left(\frac{p_1}{I_1}\right) - \left(\frac{p_2}{I_2}\right) \qquad (12)$$

$$\dot{q}_7 = f_7 = r_1r_2\left(\frac{p_1}{I_1}\right) - \left(\frac{p_3}{I_3}\right) \qquad (13)$$

$$\dot{q}_8 = f_8 = r_1r_3\left(\frac{p_1}{I_1}\right) - \left(\frac{p_4}{I_4}\right) \qquad (14)$$

$$\dot{q}_9 = f_9 = r_1r_4\left(\frac{p_1}{I_1}\right) - \left(\frac{p_5}{I_5}\right) \qquad (15)$$

Equations (7) to (15) are first order differential equations of the systems. The system of equations being linear they can be expressed in matrix form.

$$\begin{bmatrix} \dot{p}_1 \\ \dot{p}_2 \\ \dot{p}_3 \\ \dot{p}_4 \\ \dot{p}_5 \\ \dot{q}_6 \\ \dot{q}_7 \\ \dot{q}_8 \\ \dot{q}_9 \end{bmatrix} = \begin{bmatrix} -R\left(1+r_1^2\right) & 0 & 0 & 0 & 0 & -r_1 & -r_1 r_2 & -r_1 r_3 & -r_1 r_4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ r_1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ r_1 r_2 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ r_1 r_3 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ r_1 r_4 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \frac{p_1}{I_1} \\ \frac{p_2}{I_2} \\ \frac{p_3}{I_3} \\ \frac{p_4}{I_4} \\ \frac{p_5}{I_5} \\ k_6 q_6 \\ k_7 q_7 \\ k_8 q_8 \\ k_9 q_9 \end{Bmatrix} + \begin{bmatrix} F_1(t)+r_1 F_2(t) \\ 0 \\ 0 \\ -m_4 g \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (16)$$

4. Conclusion

In this paper a systematic procedure for obtaining a state space formulation in closed form for planar mechanisms is demonstrated. The procedure is based on bond graph description. The differential causalities can be eliminated by using pads and state space formulation is obtained from the bond graph.

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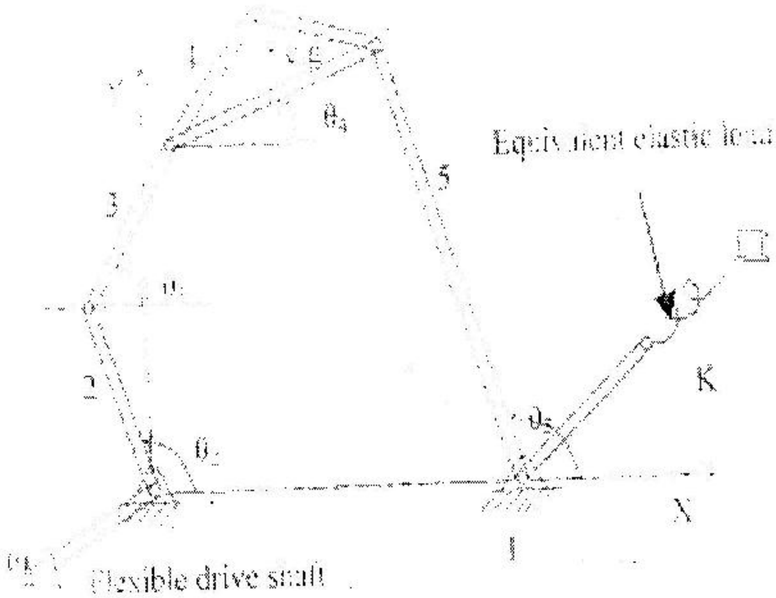

Fig. 1. Schematicof 5-bar mechanism

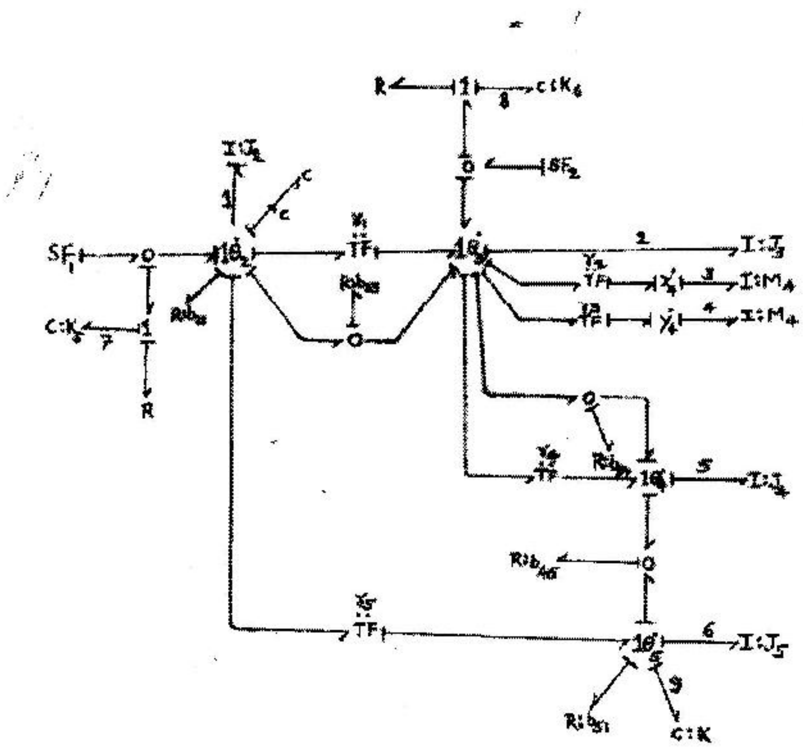

Fig. 2. Bond graph of 5-bar mechanism in fig. 1

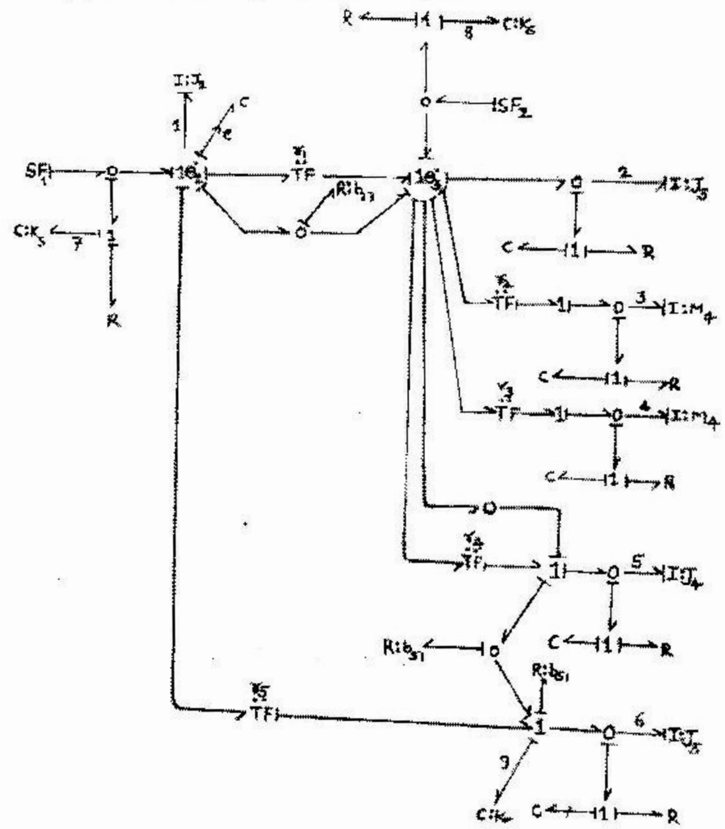

Fig. 3. Bond graph with pads of five bar mechanism in fig. 1

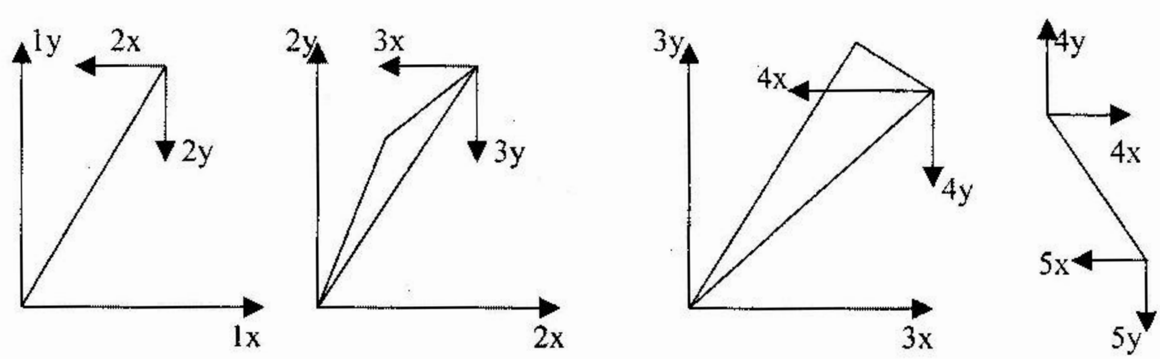

Fig. 4. (a) Free Body Diagram

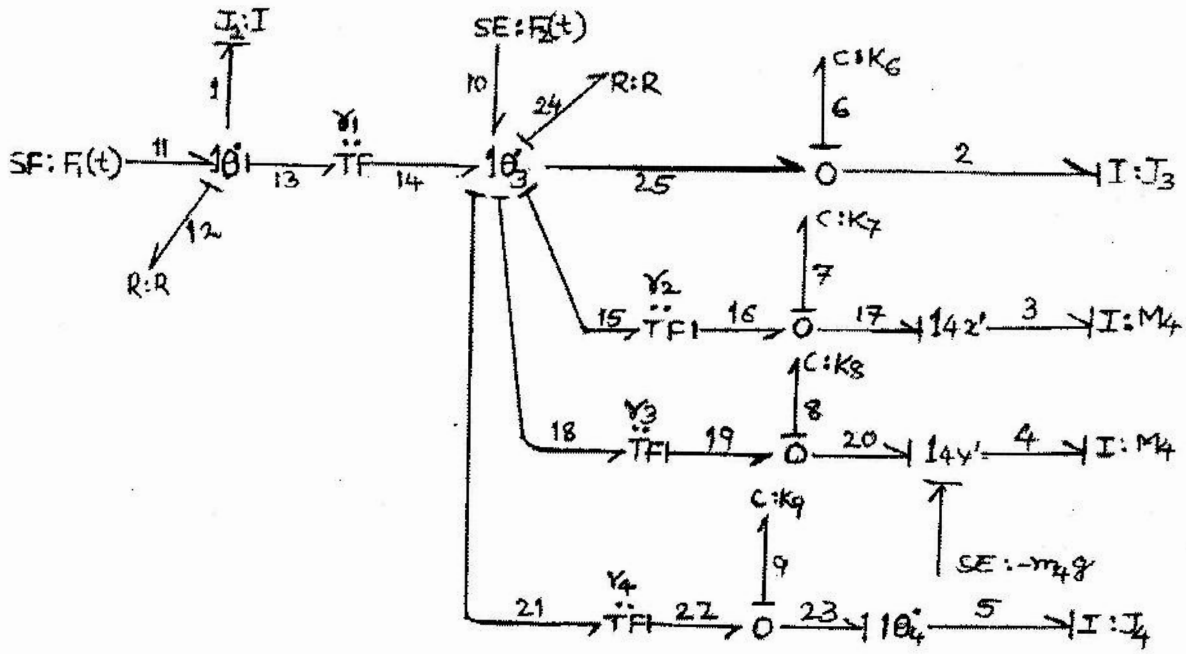

Fig. 4.(b) Bond graph of five bar mechanism with artificial compliance of fig. 1

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