# Correlating Fractal Dimension with Statistical Roughness Parameters for Turned Surface Topography

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**Abstract**: Surface roughness plays an important role in product quality and manufacturing process planning. Surface topography is generally characterized by statistical parameters such as centerline average  $(R_{\sigma})$ , root mean square average  $(R_{\sigma})$ , mean line peak spacing  $(R_{\sigma})$  etc. Since surface topography is non-stationary and multi-scaled, these parameters are not sufficient to describe the characteristics of the surface. Fractal dimension, on the other hand, describes surface roughness invariant with length scale and fractals can be extremely useful when applied to tribology .Obtaining fractal descriptions of engineering surfaces requires surface topography information to be measured, digitized and processed. The present work describes the evaluation of the fractal dimensions of the surface profiles generated by turning with different machining conditions such as depth of cut, work-piece speed and feed rate. The measured roughness parameters and fractal dimension are analyzed using ANOVA to fmd the significant machining parameters. Also, regression analysis is carried out to correlate fractal dimension with conventional roughness parameters.

Keywords: Roughness, Fractal, Turning, ANOVA, Regression

## INTRODUCTION

Turning is one of the common machining processes. In today's production, fmishing of components by turning is an important challenge due to the fact that turning has the great potential to replace other machining processes and to achieve significant reduction in production time and cost. The acceptance of tuning as a fmishing process is connected with a high form and size accuracy, high surface finish and surface integrity of the workpiece. However, in a metal removal process such as turning, the surface generated consists of inherent irregularities left by tool, which are commonly defined as surface roughness. Such a surface is composed of a large number of length scales of superimposed roughness that are generally characterized by the standard deviation of surface peaks. Three statistical characteristics are generally used to describe the microstructure of machined surface topography: texture, waviness and roughness. The texture determines the anisotropic property of the surface. The waviness reflects the reference profile (or surface). The surface roughness is formed by the micro deformation during the machining process. Conventionally, three different types of parameters, viz., amplitude parameters, spacing parameters and hybrid parameters are used to characterize surface topography in general. Amplitude parameters are measures of the vertical characteristics of the surface deviations and examples of such parameters are centre line average roughness, root mean square roughness, skewness, kurtosis, peak-tovalley height etc. Spacing parameters are measures of the horizontal characteristics of the surface deviations and examples of such parameters are mean line peak spacing, high spot count, peak count etc. On the other hand, hybrid parameters are a combination of both the vertical and horizontal characteristics of the surface deviations and examples of such parameters are root mean square slope of profile, root mean square wavelength, core roughness depth, reduced peak height, valley depth, material ratio, peak area, valley area etc. Commonly used roughness parameters are centre line average roughness, root mean square roughness, mean peak line spacing, skewness and kurtosis.

Observations show that the deviation of a surface from its mean plane is a non-stationary random process [1]. Due to the multi-scale nature of the surface, the variances and derivatives of surface peaks and other roughness parameters strongly depend on the resolution and the filter processing of measuring instruments. Ideally, rough surfaces should be characterized in such a way that the structural information

of roughness at all scales is retained. To do so, quantifying the multi-scale nature of surface roughness is essential. A unique property of a rough surface can be obtained by its scale-independent measurement. The similarity of a surface profile under different magnifications can be statistically characterized by fractal geometry since its topography is statistically self-affine. The ability to characterize surface roughness using scale-independent parameters is a specific feature of fractal approach. Fractal analysis provides information of the roughness at all length scales that exhibit fractal behavior. Based on Mandelbrot's work [2, 3], many researchers have attempted to describe and model rough surfaces using the fractal geometry theory [4-21]. Substantial investigation indicates that the surface topography has self-affme fractal characteristics [3,6,9, 12]. Initially, Gagnepain and Roques-Carmes [4] approached the 3- D roughness surface using random walk noise and white noise. The fractal dimension was calculated using the Box counting method. Majumdar and Bhushan [5] simulated the machining surface based on the modified Weierstrass-Mandelbrot function, which is called the Majumdar- Bhushan function. Majumdar and Bhushan [5] thought that the self-affine surface could be characterised using two fractal parameters; namely the fractal dimension and the amplitude coefficient. Based on the fractal characteristics of a random Cantor set, Thomas and Krajcinovic [13] established a new model for elastic-perfectly plastic contact between surfaces. Until now, many fractal models have been developed and applied for surface simulation [6], elastic-plastic contact [7, 13], tribology [8], surface texture [9], adhesion [15], friction [16], wear [10, 11,17] and so on.

In a material removal process, mechanical intervention happens over length scales, which extend from atomic dimensions to centimeters. The machine vibration, clearances and tolerances affect the outcome of the process at the largest of length scales (above  $10^3 \, \mathrm{m}$ ). The tool form, feed rate, tool radius in the case of single point cutting [22] and grit size in multiple point cutting [23], affect the process outcome at the intermediate length scales ( $10^6 \, \mathrm{to} \, 10^3 \, \mathrm{m}$ ). The roughness of the tool or details of the grit surfaces influence the fmal topography of the generated surface at the lbwest length scales ( $10^9 \, \mathrm{to} \, 10^6 \, \mathrm{m}$ ). It has been shown that surfaces formed by electric discharge machining [21], cutting or grinding [9,24], and worn surfaces [10, 11,25] have fractal structures, and fractal parameters can reflect the intrinsic properties of surfaces to overcome the disadvantages of conventional roughness parameters. However, there is lack of information regarding the characteristics of roughness generated in turning particularly with respect to fractal dimension. Thus there is scope and need for further study in this respect. In the present work, the surface profiles generated by turning of mild steel are measured, digitized and processed to evaluate the statistical roughness parameters and fractal dimension. The relation between the fractal dimension and statistical roughness parameters are also investigated.

# SCOPE AND EXPERIMENTAL METHODOLOGY

The main aim of the present study was evaluation of surface finish in turning and correlation of different roughness parameters with fractal dimension for varying cutting conditions. Turning tests were carried out on a conventional lathe (Make -Mysore Kirloskar Ltd.). Workpiece material is mild steel (C45 medium carbon steel equivalent to AISI 1045 grade). The workpiece employed were in the form of cylindrical bars about 50 mm long with an external diameter of 40 mm. During machining, no chatter, which would create significant vibrations, was identified. The workpiece speeds (rpm) chosen were 270,421,646 and 1000. Feed rates (mm/rev) selected were 0.028, 0.056,0.084 and 0.113. Depths of cut (mm) taken were 0.10,0.15, 0.20 and 0.25. Thus total 64 workpiece were turned for varying combinations of workpiece speed, feed rate and depth of cut. The surface roughness parameters on the generated-turned surfaces were measured with a portable stylus-type profilometer (Surtronic 3+, Taylor Hobson) equipped with a diamond stylus having tip radius 5µm. The profilometer was set to a cut-off length of 0.8 mm. The measurements on a single sample surface were repeated four times and the average value has been used as a data point. The measured profile is digitized and processed through the dedicated advanced surface finiish analysis software Talyprofile for evaluation of the roughness parameters and fractal dimension. In this software fractal analysis is done using structure function method. Parameters used in the present investigation are: Vertical parameters -  $R_a$  (CLA,  $\mu$ m) and  $R_a$  (RMS,  $\mu$ m), Transverse parameters -  $R_{s\eta}$ (mean line peak spacing,  $\mu$ m), Peak shape and amplitude parameters -  $R_{sk}$  (skewness),  $R_{ku}$  (kurtosis) and Fractal parameter - D (fractal dimension).

## **ANALYSIS**

In the present study surfaces are generated by turning of mild steel by varying the machining parameters with the motivation to study the effect of these machining parameters on surface roughness produced. The appropriate procedure for testing the significance of several parameters is the analysis of variance (ANOVA). The measured roughness parameters and fractal dimension are analyzed using ANOVA. Also, regression analysis is carried out to correlate fractal dimension with conventional roughness parameters.

## **ANOVA Test**

To determine the significant factors or factor interactions in affecting the surface roughness, measured roughness parameters and fractal dimension are tested in analysis of variance (ANOVA). The analysis is done with the help of MINITAB software (Release 13.1). ANOVA tables, main effects plots and full interaction plots are prepared for each parameter. Figures 1 and 2 show such typical main effects plot and interaction plot for fractal dimension. Such plots are prepared for each of the roughness parameters, but for the sake of brevity all such plots are not shown here. Actually, the main effect plot and the interaction plot appear to support each other for conclusion reached from ANOVA table. The ANOVA test is carried out based on 95% confidence level. Based on 95% confidence interval, RPM and feed rate have a statistically significant impact on fractal dimension, since their p-values are found to be smaller than 5% though their interaction has no significant effect on fractal dimension. Only RPM is the significant factor in determining the center line average value ( $R_a$ ). For root mean square roughness value ( $R_a$ ), it is found that RPM and interaction of RPM and feed rate are the significant factors. For skewness, RPM and feed rate are the significant factors. For kurtosis, RPM, feed rate and their interaction are the significant factors. Mean line peak spacing very much depends on feed rate and its interaction with RPM. Depth of cut has no statistical significant effect on roughness parameters and fractal dimension.

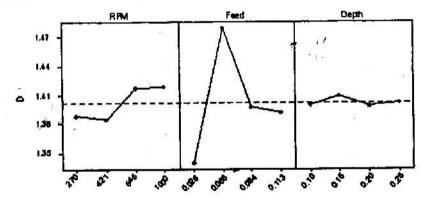


Fig.1 Main Effects Plot for D

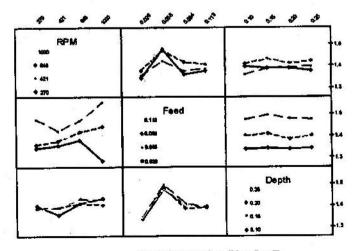
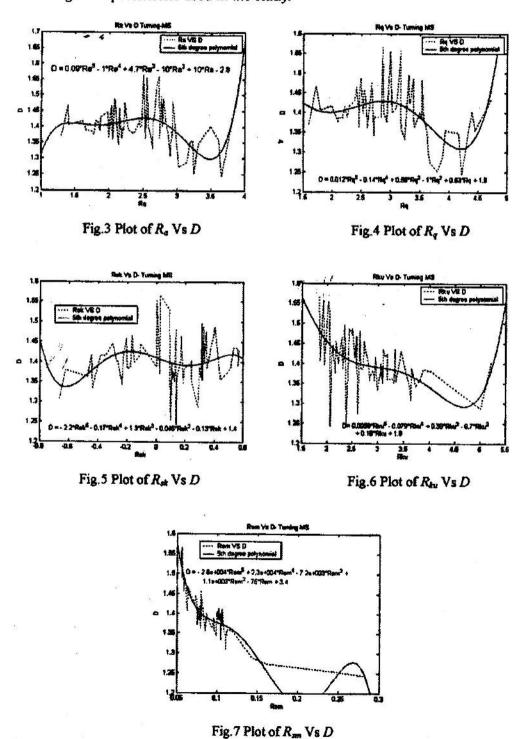


Fig.2 Interaction Plot for D

# Regression Analysis

In any experiment, the experimenter is frequently interested in developing an interpolation equation for the response variable in the experiment. These equations are empirical models of the process that has been studied. The general approach to fitting empirical models is called regression analysis. In the present study, the measured values of conventional roughness parameters and fractal dimension for different processes and materials are correlated by non-linear regression analysis using filth order polynomial model. The analysis has been carried out using MA TLAB 6.5 as the computational platform. Figures 3-7 show the regression analysis results for different conventional roughness parameters with fractal dimension. The following empirical equations are obtained for the relation between fractal dimension and conventional roughness parameters used in the study.



$$D = 0.09 * R_a^5 - R_a^4 + 4.7 * R_a^3 - 10 * R_a^2 + 10 * R_a - 2.8$$

$$D = 0.012 * R_q^5 - 0.14 * R_q^4 + 0.59 * R_q^3 - R_q^2 + 0.53 * R_q + 1.5$$

$$D = -2.2 * R_{sk}^5 - 0.17 * R_{sk}^4 + 1.3 * R_{sk}^3 - 0.045 * R_{sk}^2 - 0.13 * R_{sk} + 1.4$$

$$D = 0.0059 * R_{tu}^5 - 0.079 * R_{tu}^4 + 0.38 * R_{tu}^3 - 0.7 R_{tu}^2 + 0.18 * R_{tu} + 1.9$$

$$D = -2.8 * 10^4 * R_{sk}^5 + 2.3 * 10^4 * R_{sk}^4 - 7.2 * 10^3 * R_{sk}^3 - 1.1 * 10^3 * R_{sk}^2 + 75 * R_{sk}^3 + 3.4$$

#### CONCLUSIONS

In case of turning of mild steel, RPM and feed rate are the main factors affecting the roughness parameters.  $R_a$  and  $R_a$  values greatly depend on RPM. At higher values of RPM, smoother surface may be obtained. For most of the cases, feed rate is the main factor to affect the parameters. Fractal dimension, skewness and kurtosis are controlled by both RPM and feed rate. It is observed that with increasing RPM gives better surface finish. In the present study, the measured values of conventional roughness parameters and fractal dimension for varying cutting conditions are correlated by non-linear regression analysis using fifth order polynomial model. However, more study is needed in this respect to have some generalized relation between fractal dimension and conventional roughness parameters.

## **ACKNOWLEDGEMENTS**

The second author gratefully acknowledges the fmancial assistance of DST, Govt. of India through a SERC Fast Track Project for young scientists vide Ref. No. SR/ FTP/ ETA- 11/2004 dated 28.06.2004 and useful discussions with Dr.A. Bandyopadhyay during experiments.

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