Detection of Isomorphism Using Eigenvalues and Eigenvectors

S.F.PATIL*
Asst. Prof., Department of Mechanical Engineering,
K L E Society's College of Engineering and Technology, Udyambag, BELGAUM-8

S.C. PILLI
Principal, K L E Society's College of Engineering and Technology, Udyambag, BELGAUM-8.

K. VIJAYANANDA
QUEST India, Bangalore.

Abstract:

In the literature various methods are presented to identify distinct kinematic chains, mechanisms with N links ($4 \leq N \leq 16$) and F freedoms ($1 \leq F \leq 8$). However, no method is fool proof, there are counter examples for each of the method. In this paper, the eigen values and eigen vectors are used to identify the isomorphic chains. This method is elegant and strait forward to determine the distinct kinematic chains.

1.0 Introduction:

A major problem frequently encountered in structural synthesis of kinematic chains is that of detecting possible structural isomorphism between two given chains. Many attempts[1-18] have been made in literature to develop reliable and computationally efficient tests for isomorphism. These tests can be grouped under four groups namely: 1) characteristic polynomial based approaches, 2) code-based approaches, 3) Hamming-number based approaches, and 4) distance or path based approaches. The studies in literature identify distinct kinematic chains, mechanisms with N links ($4 \leq N \leq 16$) and F freedoms ($1 \leq F \leq 8$) [1].

Uicker and Raicu [2] seem to be the first researchers who have investigated the properties of the characteristic polynomial of the adjacency matrix of a kinematic chain. Murthyunjaya and Raghavan [3] applied Bocher's formula for the determination of the characteristic coefficients and presented a counter example for the uniqueness of the characteristic polynomial and showed that polynomial is unique for closed and connected kinematic graphs. Yan and Hall[4,5] presented rules and theorems by which characteristic polynomial of kinematic chain and its coefficients are determined. Mruthyunjaya and Balasubramanian [6] worked on characteristic polynomial of a vertex-vertex degree matrix, they brought light on counter examples. Dubey and Rao[7] considered characteristic polynomial of distance matrix. Ambekar and Agrawal[8] proposed max code and min code methods for the detection of isomorphism. Kim and Kawk [10] proposed heuristic algorithm that uniquely labels the links of a chain which leads to a unique code. Shin and Krishnmurthy[11] presented the standard code theory for the detection of isomorphism. Rao[12] introduced the concept of Hamming distances from information and communication theory to the study of kinematic structure. Rao and Varadaraju[13] defined link Hamming string as an index for testing isomorphism. Rao[14] illustrated a method by using chain Hamming matrix by which reliability of isomorphism test based on primary Hamming string is increased. Yan and Hwang[15] defined the linkage path code of a kinematic chain. Yadav et.al.[16] presented a sequential three-step test for isomorphism. Vijayananda[17] has developed a new isomorphism test based on the visual description of a chain and is suitable for computer implementation. Shende and Rao[18] proposed a method based on summation polynomials.

2.0 Eigen values and eigen vectors

The eigen values and eigen vectors[19,20] are useful in many problems including the solution of

* *Corresponding author: patil_sf@yahoo.com*

systems of linear differential equations. Eigen values can have important physical significance in many problems. They represent the natural frequencies of vibration of a mechanical system or the fundamental frequencies of oscillation in certain electrical systems. One of the method of determining the eigen values involves solving a polynomial equation that may have complex roots. Hence, the eigen values and eigen vectors can be used to identify the isomorphic chains.

The best numerical methods used to find eigen values avoid the characteristic polynomial entirely. MATLAB finds the characteristic polynomial of a matrix A by first computing the egien values $\lambda_1, \ldots \lambda_n$ of A and then expanding the product $(\lambda - \lambda_1)(\lambda - \lambda_2) \ldots (\lambda - \lambda_n)$.

An eigen vector of an n x n matrix A is a non zero vector. Such that $A_x = \lambda x$ for some scalar λ. A scalar λ is called an eigen value of A if there is a nontrivial solution x : $Ax = \lambda x$; x is called eigen vector corresponding to λ.

$$Ax - \lambda x = (A - \lambda I)x = 0 \quad \ldots\ldots (1)$$

Eqn.(1) has a nontrivial solution if and only if the determinant of $(A - \lambda I)$ is zero.

$$|A - \lambda I| = 0 \quad \ldots\ldots (2)$$

Eqn (2) can be expanded in the form of polynomial in λ as in eqn. (3).

$$C(\lambda) = (-\lambda)^n + C_{n-1}(-\lambda)^{n-1} + \ldots\ldots\ldots\ldots + C_1(-\lambda) + C_0 = 0 \quad \ldots\ldots (3)$$

Eqn. (3) is known as the characteristic polynomial of the matrix A, which can be factored with n roots as in eqn. (4).

$$C(\lambda) = (\lambda - \lambda_1)(\lambda - \lambda_2)(\lambda - \lambda_3) \ldots\ldots\ldots\ldots (\lambda - \lambda_n) = \prod_{i=1}^{n} (\lambda - \lambda_i) \quad \ldots\ldots (4)$$

If λ is an eigen value of 'A', any nontrivial solution to $(A - \lambda I)x = 0$ is an eigen vector of 'A' corresponding eigen value λ.

Eigen vectors corresponding to the distinct eigen values of a matrix are linearly independent. If an n x n matrix has m distinct eigen values ($m \leq n$) then the matrix has at least m linearly independent eigen vectors.

3.0 Detection of isomorphism in kinematic chains:

The link-link adjacency matrix [A] of a kinematic chain is adopted in detecting the isomorphism using eigen values and eigen vector. MATLAB ver 7 is used to evaluate the eigen values and eigen vectors. If eigen values and eigen vectors are identical, the chains are isomorphic.

4.0 Results.

Fig. 1 shows two 10 link kinematic chains. Fig. 2 shows the link-link adjacency matrices of the chains shown in Fig. 1. The chain-Hamming strings are identical. Hence, they were classified as isomorphic[13]. However, on use of secondary chain Hamming strings which differ in the values indicates that the kinematic chains are nonisomorphic[13]. The eigen values of the two adjacency matrices sorted in the order of magnitude are $[-2.5616, -2.0, -1.4142, -0.7321, 0.0, 0.0, 1.0, 1.4142, 1.5616, 2.7321]^T$ and $[-2.7321, -1.4142, -1.4142, -1.0, -0.7321, 0.7321, 1.0, 1.4142, 1.4142, 2.7321]^T$. Since, the eigen values are distinct it can be concluded that the two kinematic chains shown in Fig. 1 are nonisomorphic.

Fi.g. 3 shows two 10 link kinematic chains. Fig. 4 shows the link-link adjacency matrices of the chains shown in Fig. 3. The Arranged Sequence of the Total Distance Ranks of all the Joints(ASTDRJs) are identical. Hence, they were classified as isomorphic[16]. The eigen values of the two adjacency matrices sorted in the order of magnitude are $[-2.4142, -2.1701, -1.2143, -1.0, -0.3111, 0.4142, 1.0, 1.4812, 1.5392, 2.6751]^T$ and $[-2.5616, -2.3429, -0.7321, -0.4707, 0.0, 0.0, 0.0, 1.5616, 1.8136, 2.7321]^T$. Since, the eigen values are distinct it can be concluded that the two kinematic chains shown in Fig. 3 are nonisomorphic.

5.0 Conclusions :

The paper presents a simple and straight forward method of using eigen values and eigen vector to identify the isomorphic kinematic chains. This is also suitable for computer implementation. Some case study results are presented to illustrate the method.

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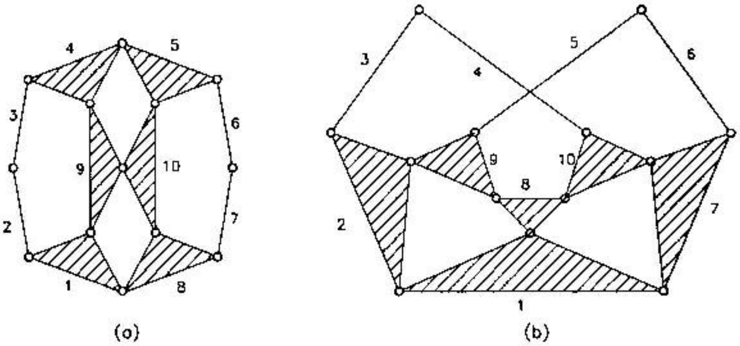

Fig.1 10 link, single degree of freedom, nonisomorphic kinematic chains

$$A1=\begin{bmatrix} 0&1&0&0&0&0&0&1&1&0\\ 1&0&1&0&0&0&0&0&0&0\\ 0&1&0&1&0&0&0&0&0&0\\ 0&0&1&0&1&0&0&0&1&0\\ 0&0&0&1&0&1&0&0&0&1\\ 0&0&0&0&1&0&1&0&0&0\\ 0&0&0&0&0&1&0&1&0&0\\ 1&0&0&0&0&0&1&0&0&1\\ 1&0&0&1&0&0&0&0&0&1\\ 0&0&0&0&1&0&0&1&1&0 \end{bmatrix} \qquad A2=\begin{bmatrix} 0&1&0&0&0&0&1&1&0&0\\ 1&0&1&0&0&0&0&0&1&0\\ 0&1&0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&1\\ 0&0&0&0&0&1&0&0&1&0\\ 0&0&0&0&1&0&1&0&0&0\\ 1&0&0&0&0&1&0&0&0&1\\ 1&0&0&0&0&0&0&0&1&1\\ 0&1&0&0&1&0&0&1&0&0\\ 0&0&0&1&0&0&1&1&0&0 \end{bmatrix}$$

Fig.2 Adjancency matrices of kinematic chains shown in Fig.1

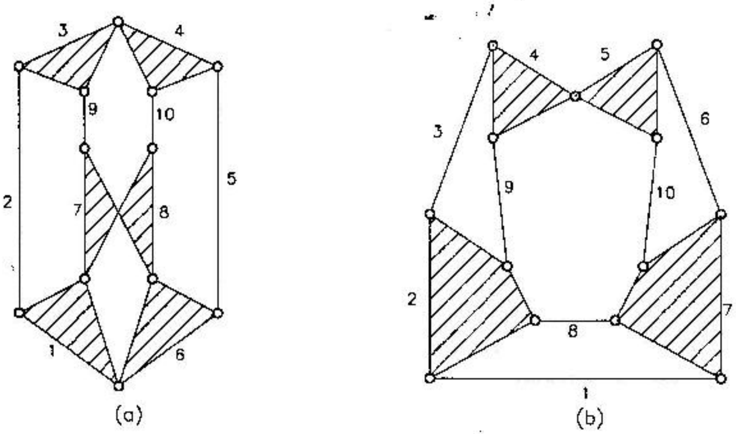

Fig. 3 10 link, single degree of freedom, nonisomorphic kinematic chains

$$A1=\begin{bmatrix} 0&1&0&0&0&1&1&0&0&0\\ 1&0&1&0&0&0&0&0&0&0\\ 0&1&0&1&0&0&0&0&1&0\\ 0&0&1&0&1&0&0&0&0&1\\ 0&0&0&1&0&1&0&0&0&0\\ 1&0&0&0&1&0&0&1&0&0\\ 1&0&0&0&0&0&0&1&1&0\\ 0&0&0&0&0&1&1&0&0&1\\ 0&0&1&0&0&0&1&0&0&0\\ 0&0&0&1&0&0&0&1&0&0 \end{bmatrix} \qquad A2=\begin{bmatrix} 0&1&0&0&0&0&1&0&0&0\\ 1&0&1&0&0&0&0&1&1&0\\ 0&1&0&1&0&0&0&0&0&0\\ 0&0&1&0&1&0&0&0&1&0\\ 0&0&0&1&0&1&0&0&0&1\\ 0&0&0&0&1&0&1&0&0&0\\ 1&0&0&0&0&1&0&1&0&1\\ 0&1&0&0&0&0&1&0&0&0\\ 0&1&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&1&0&0&0 \end{bmatrix}$$

Fig.4 Adjancency matrices of kinematic chains shown in Fig.3