Trajectory Selection for Robotic Applications

N.P.S. Deo and S.K. Saha*
Lecturer, Department of Mechanical Engineering
BBSB Engineering College, Fatehgarh-Sahib
Punjab, India, PIN-140407
E-mail: deo_nps@yahoo.com
*Department of Mechanical Engineering, Indian Institute of Technology Delhi, New Delhi

ABSTRACT

Finding a smooth trajectory to move an object is necessary for object manipulation in robotics and automation. Since many alternative trajectories are possible to perform a task, an evaluation scheme to suggest the best trajectory for a given application becomes crucial. An evaluation technique of trajectories based on some performance characteristics of trajectories, viz. minimum travel time, path length of a robot end-effector between its initial and final configurations, RMS (Root Mean Square) value of joint accelerations, RMS value of joint torques and total work done for the given motion, is presented in this paper. Upon specification of a robot geometry, maximum speed of each joint, description of a desired motion, and a set of trajectories, a systematic evaluation of each trajectory is performed to determine the optimal trajectory for the given robot.

1. Introduction

So far, research effort in trajectory planning has been directed at creating new trajectory generation algorithms by incorporating different constraint criteria. Various trajectory parameters have been optimized in the trajectory generation process so that optimization is being achieved at the design stage. However, very little attention is directed towards the evaluation of the trajectories once they have been created. So, optimization at the implementation stage has been neglected.

A number of techniques have been developed for the planning of minimum time trajectories of industrial robots under the constraints of joint velocities, accelerations, jerks, and torques [1, 2, 3]. Alternatively, Nnaji and Asano [4] developed an evaluation technique based on joint torque, work and power requirements. Five trajectories were evaluated for eight different classes of robots, where power and torque have been used as constraints rather than evaluation criteria. The best trajectory was determined by choosing the one that meets the torque and power constraints of the joint drives and has the minimum value. Macfarlane and Croft [5] also developed a method using the concatenation of a fifth-order polynomial to provide a smooth, controlled, near time optimal trajectory for the point-to-point motion with jerk limits. A simple straight line point-to-point motion in the Cartesian space was chosen to compare the simulated motion times of three different types of trajectories, namely Linear Segment with Parabolic Blends (LSPB), quintic concatenation, and a single quintic.

This paper is organized as follows; Section 2 introduces the types of trajectories and the proposed criteria for trajectory selection; Section 3 explains the evaluation algorithm and methods used for the ranking of trajectories; Section 4 illustrates the selection criteria for the ranking of trajectories using the example of a two degree of freedom planar manipulator. Finally, conclusions are given in Section 5.

2. Trajectories

The space curve, which a robot hand moves along from an initial to final location, is called the path or trajectory. The trajectory planning involves interpolation or approximation of a path by any smooth function (a function is considered smooth, if it is continuous and has at least continuous first derivative).

It generates a sequence of time-based control set points for the control of robot from the initial to final location. These time-based set points when expressed in the joint space denote the time-history of position, velocity, and acceleration for each degree of freedom of the joints. Such trajectories are referred as the joint space trajectories, in contrast to the Cartesian space trajectories where the positions and orientations of the robot end-effectors and their time derivatives are specified [6].

2.1 Types of trajectory

Selection of trajectories depends on the type of end-effector motion required. In this study, we are considering point-to-point motion of the end-effector without any obstacles in the path between initial and final configurations. Five functions, as commonly used in the literature [1, 4, 6], are chosen here for the evaluation purposes. They are given in Table 1.

Table 1 Types of trajectories

Sr. No.	Name	Function
1	Bang-bang	$\theta(\tau) = 2\tau^2$ for $(0 \le \tau \le 0.5)$ $= -1+4\tau - 2\tau^2$ for $(0.5 \le \tau \le 1.0)$
2	Cubic Polynomial	$\theta(\tau) = -2\tau^3 + 3\tau^2$
3	Quintic Polynomial	$\theta(\tau) = 6\tau^5 - 15\tau^4 + 10\tau^3$
4	Cycloidal	$\theta(\tau) = t - 1/2\pi \sin 2\pi\tau$
5	Cosine	$\theta(\tau) = 0.5(1- \cos\pi\tau)$

In Table 1, τ and θ are the normalized time and the end-effector displacement, respectively, such that $0 \le \theta \le 1$; $0 \le \tau \le 1$; and $\tau = t/T$ — T being the total time of operation. If θ_j^I and θ_j^F are the given initial and final values of the jth joint variable, then we can represent each of the joint variable θ_j through its range of motion as [1]

$$\theta_j(t) = \theta_j^I + \theta_j^R \theta(\tau), \text{ where } \theta_j^R = \theta_j^F - \theta_j^I \tag{1}$$

In vector form this equation can be written as,

$$\theta(t) = \theta^I + \theta^R\, \theta(\tau), \text{ where } \theta^R = \theta^F - \theta^I \tag{2}$$

and hence,

$$\dot{\theta}(t) = \frac{1}{T}\theta^R\, \theta'(\tau),\ \ddot{\theta}(t) = \frac{1}{T^2}\theta^R\, \theta''(\tau), \text{ and } \dddot{\theta}(t) = \frac{1}{T^3}\theta^R\, \theta'''(\tau) \tag{3}$$

where $\dot{\theta}, \ddot{\theta}$ and $\dddot{\theta}$ represent the first, second and third time derivatives of $\theta = [\theta_1, \ldots, \theta_n]^T$.

2.2 Evaluation Criteria

Proper identification of evaluation criteria is critically important while comparing various trajectories. For this study, five trajectory parameters are selected based on those reported in the literature, e.g., in [1, 4], and others for the purpose of evaluation. They are explained below.

1. *Minimum motion time*: Minimum motion time for each joint satisfying maximum velocity constraint, $\dot{\theta}_j \le \dot{\theta}_j^{max}$, is obtained from (3) as

$$T_{min} = \theta_j^R\, \theta'(\tau) / \dot{\theta}_j^{max} \tag{4}$$

The minimum time for overall motion is the largest value of the minimum times taken by all the joints in a robot.

2. RMS acceleration: Once the minimum motion time is known, maximum acceleration is calculated for all joints. RMS value of the accelerations is then calculated for each trajectory using the formula

$$a_{RMS} = \sqrt{\frac{a_{1,\max}^2 + \ldots + a_{n,\max}^2}{n}} \tag{5}$$

where n represents the number of joints.

3. RMS torque: Torque for each joint is found using Newton-Euler formulation [6] at every time step. Then, RMS value of maximum torques is calculated from the following formula:

$$\phi_{RMS} = \sqrt{\frac{\phi_{1,\max}^2 + \ldots + \phi_{n,\max}^2}{n}} \tag{6}$$

where ϕ denotes torque and $\phi_{i,max}$ is the maximum torque at the ith joint during the end-effector motion along a given trajectory.

4. Work done: Once torque is known at every time step, work done by the actuator in that step is found by multiplying the absolute value of the torque with the joint displacement in that step. If we assume that time step is never large enough to violate the continuous nature of the torque, the total work done by the actuator can be found as

$$W_b = \sum_{i=1}^{b} \left(\phi_i^1 \cdot \Delta\theta_i^1 + \ldots + \phi_i^n \cdot \Delta\theta_i^n \right) \tag{7}$$

where b is the number of time steps.

5. Path length: Path length of a small segment of the trajectory traced by the end-effector, Δs, is found by approximating it with a straight line. If acceleration is assumed constant during the segment, it can be calculated as

$$\Delta s = \frac{v^2 - u^2}{2a} \tag{8}$$

where v and u are respectively the linear velocity of end-effector at the start and end of each path segment, whereas a is constant acceleration during the path segment. Now, the total path length is calculated by simply adding the individual path lengths for the complete motion, i.e.,

$$s = \sum_{i=1}^{b} \Delta s_i \tag{9}$$

where b is the number of steps, as defined after eq. (7).

3. Evaluation Procedure

A computer program in C++ was written to find out the values for the evaluation criteria of all trajectories. The algorithm is summarized as follows.

1. Read initial and final configurations in joint space, maximum speed of each joint, and the no. of steps to complete the motion;
2. Calculate minimum motion time from eq. (4) for each trajectory;

3. Calculate maximum acceleration of each joint from eq. (3), and then the RMS value from equation (5);
4. Read link lengths, link masses, moment of inertia of each link, distance of center of mass for each link from one of its joints, and the acceleration due to gravity if the motion is not in the horizontal plane;
5. Calculate joint torques at every time step, and then the RMS value of maximum joint torques from eq. (6);
6. Calculate total work done using equation (7);
7. Calculate path length of the end-effector from eq. (9).

Once the values for all trajectories are known, we can rank the trajectories with respect to each parameter. To find out an overall ranking of trajectories we have used two methods, lowest parameters sum and the MADM (Multiple Attribute Decision Making) [8], as explained next.

3.1 Lowest parameter sum method

This is a simple approach for overall ranking of trajectories. As values have different units and magnitudes, they cannot be processed and compared as they are for the evaluation criteria. So, normalization of the same is done by assigning the least value of each parameter as unity, and dividing other values by this value to get the relative weights. Now, the sum of normalized parameters for each trajectory is calculated and ranked. The best trajectory is the one having least sum.

3.2 MADM approach

The steps to determine the weights and subsequently ranking the trajectories using TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) approach of MADM [8] are as follows:

1. Represent information about evaluation criteria values of various trajectories in the form of a matrix. Such a matrix is called as Decision Matrix, **D**. Each row of this matrix is allocated to one candidate trajectory, and each column to one evaluation criterion under consideration. Therefore an element dij of the decision matrix **D** gives the value of the *j*th criterion in the raw form (non-normalized). Thus, if the number of trajectories is '*m*' and the number of parameters is '*q*', the decision matrix **D**, is an *m* x *q* matrix.

2. Obtain information from the user or the group of experts on the relative importance of attributes and construct the relative importance matrix **A**. This will be an *q* x *q* matrix of which the symmetric terms will be reciprocals of each other.

3. Obtain maximum eigenvalue λ and associated eigenvector as weight vector **w**, where w_i represents the weight of the ith attribute.

4. Construct the normalized decision matrix, **N** using

$$n_{ij} = \frac{d_{ij}}{\sqrt{\left(\sum_{i=1}^{m} d_{ij}^{2}\right)}} \tag{10}$$

where n_{ij} is an element of the normalized decision matrix, and d_{ij} is the numerical outcome of the i^{th} option with respect to the j^{th} criterion.

5. Determine the weighted normalized decision matrix, **V**, as

$$V = \begin{bmatrix} w_1 n_{1,1} & w_2 n_{2,2} & \cdots & w_n n_{1,n} \\ w_1 n_{2,1} & \ddots & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ w_1 n_{m,1} & w_2 n_{m,2} & \cdots & w_n n_{m,n} \end{bmatrix} = \begin{bmatrix} v_{1,1} & v_{1,2} & \cdots & v_{1,n} \\ v_{2,1} & \ddots & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ v_{m,1} & v_{m,2} & \cdots & v_{m,n} \end{bmatrix} \tag{11}$$

6. Using **V** determine the +ve and -ve benchmark trajectories, where both benchmark trajectories are hypothetical trajectories. We consider the +ve benchmark trajectory as the one having best parameter values, where as the -ve benchmark trajectory as the one having worst parameter values.

3.2.1 TOPSIS method

The TOPSIS method is based on the concept that the optimum one should have the shortest distance from the +ve benchmark trajectory, and be farthest from the -ve benchmark trajectory. Here, separation measures, P^+ and P^- from the +ve and -ve benchmark trajectories are calculated as follows [8]

$$P_i^+ = \sqrt{\left[\sum_{j=1}^{q}\left(v_{ij} - v_1^+\right)^2\right]} \qquad (i = 1,2,\ldots,m) \tag{12}$$

And

$$P_i^- = \sqrt{\left[\sum_{j=1}^{q}\left(v_{ij} - v_1^-\right)^2\right]} \qquad (i = 1,2,\ldots,m) \tag{13}$$

Now, the relative closeness to the +ve benchmark trajectory, R*, can be calculated, and the trajectories can be ranked, such that, one having largest value of R*, is the best, i.e.,

$$R^* = P_i^- / (P_i^+ + P_i^+) \tag{14}$$

4. Illustrative example

A 2-DOF planar robot [7], as shown in Fig. 1, is selected for the illustration of five selection strategies proposed in this paper. Trajectories introduced in sub-section 2.1 are considered here to select the best one. Consider the following specifications:

Maximum speed of joint 1 = 1.16 rad/s; Length of link 1 = 0.5 m; Moment of inertia of link 1about its center of mass, parallel to z-axis = 0.1 kg.m^2; Mass of link 1 = 10 kg;

Distance of center of mass of link 1from joint 1 = 0.25 m; Maximum speed of joint 2= 1.57 rad/s; Length of link 2 = 0.5 m; Moment of inertia of link 2 about an axis passing through its center of mass, parallel to z-axis = 0.1kg.m^2

Mass of link 2 = 10 kg; Distance of center of mass of link 2 from joint 2 = 0.25 m

Trajectories are evaluated for the following motion of the robot:

Initial angle of joint 1 = 10°; Final angle of joint 1 = 40°; Initial angle of joint 2 = 5°; Final angle of joint 1 = 35°

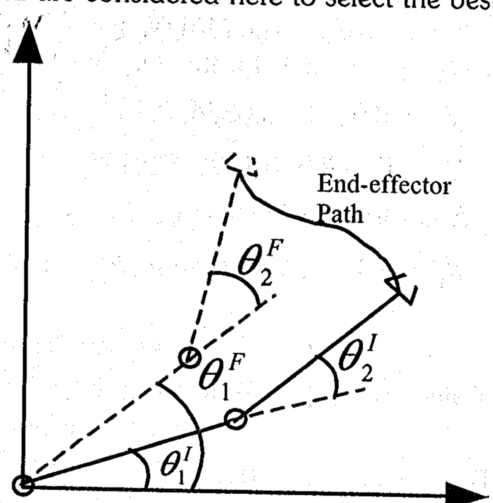

Fig. 1 A 2- D.O.F. Planar Robot

A C++ program is developed to calculate the evaluation parameters, which are used to construct the decision matrix and others to rank the trajectories. The results are shown in Table 2.

Table 2 Actual parameter values for all trajectory functions

Sr.No.	Trajectory	Tmin (sec)	Acceleration (RMS) (r/s²)	Torque (RMS) (N-m)	Path length (m)	Work done done (J)
1	Bang-bang	0.4514	15.1700	78.6425	0.6983	30.6011
2	Cubic Polynomial	0.6771	10.1133	66.4746	0.5401	32.0658
3	Quintic Polynomial	0.8463	8.6072	56.7418	0.5765	31.9521
4	Cycloidal	0.9028	5.9572	55.9171	0.5871	31.9155
5	Cosine	0.7091	7.5833	60.3656	0.5562	32.0367

Based on the lowest parameter sum method, the relative parameter values and their total for each trajectory are obtained in Table 3.

Table 3 Normalized parameter values for all trajectory functions

Trajectory	T_{min}	Acceleration (RMS)	Torque (RMS)	Path length	Work Done	Total
Bang-bang	1.0000	2.5465	1.4064	1.2929	1.0000	7.2458
Cubic	1.5000	1.6977	1.1888	1.0000	1.0479	6.4344
Quintic	1.8748	1.4448	1.0147	1.0674	1.0441	6.4458
Cycloidal	2.0000	1.0000	1.0000	1.0870	1.0430	6.1300
Cosine	1.5709	1.2730	1.0796	1.0298	1.0469	6.0002

Alternatively, using MADM and TOPSIS approach as explained in sub-section 4.2, the weighted normalized parameters for the +ve and -ve benchmark trajectories are as follows:

$$V^{+} = (0.0484, 0.0544, 0.1040, 0.0988, 0.0986) \quad (15)$$

$$V^{-} = (0.0968, 0.1386, 0.1462, 0.1277, 0.1034) \quad (16)$$

Accordingly, relative closeness to optimum trajectory of all trajectories eq. (15) is given by

$$R_1^* = 0.3304, R_2^* = 0.5636, R_3^* = 0.6059, R_4^* = 0.6609, R_5^* = 0.7186$$

Table 4 Ranking of trajectories

Trajectory	TOPSIS- Closeness to the +ve benchmark trajectory, R*	Rank based on R*	Total sum of parameters	**Rank based on total**
Bang-bang	0.3304	5	7.2458	5
Cubic	0.5636	4	6.4344	3
Quintic	0.6059	3	6.4458	4
Cycloidal	0.6609	2	6.1300	2
Cosine	0.7186	1	6.0002	1

Now, the ranking of the trajectories based on the two methodologies presented in sub-sections 3.1 and 3.2 is given in Table 4, which shows that cosine function is the best. Apparently, this means that with respect to every evaluation parameter, say, T_{min} or RMS Torque or any other, the values for the consine trajectory are not very far compared from the corresponding best trajectories. For example, Bang-bang is best in terms of T_{min}. However, in terms of the RMS Torque and Path length values, it is about 40% and

30% higher from the best, as evident from Table 2. For the cosine trajectory, on the other hand, the RMS Torque and Path length values are only 8% and 3% higher from their best values. This is justified, as what is given in Table 2 is a kind of averaging only.

5. Conclusions

Evaluation of different trajectories on the basis of five characteristics of a trajectory is done. The contributions of this work are summarized as

1. From the data generated by this work one can select an optimum trajectory for a particular robotic application when the user knows no selection criteria;
2. Ranking of trajectories can be done for a desirable trajectory characteristic;
3. Parameter values for optimum trajectory can be used at the design stage for making a robot capable of performing a task in optimum manner;
4. Work can be extended to select optimum trajectory for similar applications in case of higher degree of freedom robots also.

REFERENCES

1. J. Angeles, Fundamentals of Robotic Mechanical Systems, Second Edition, New York: Springer-Verlag, 2003.
2. H.H. Tan, R.B. Potts, "Minimum-time trajectory planner for the discrete dynamic robot model with dynamic constraints," IEEE journal of Robotics and Automation, Vol. 4, No. 2, April 1988, pp. 174-185.
3. S.A. Bazaz, B. Tondu, "On-line computing of a robotic manipulator joint trajectory with velocity and acceleration constraints," in Proceedings of the 1997 IEEE International Symposium on Assembly and Task Planning, Marina del Rey, CA, August 1997, pp. 1-6.
4. B.O. Nnaji, D.K. Asano, "Evaluation of trajectories for different classes of robots," Elsevier Journal of Robotics and Computer-Integrated Manufacturing, Vol6, Issue 1, 1989, pp. 25-35.
5. S. Macfarlane, E.A. Croft, "Jerk-Bounded Manipulator trajectory planning: Design for real-time applications," IEEE Trans. on Robotics and Automation, Vol. 19, No. 1, Feb. 2003, pp. 42-52.
6. J.J. Craig, Introduction to Robotics: Mechanics and Control, Second Edition, Pearson Education Inc., 1989.
7. Y.G. Lin, J.Y. Choi, " A simple algorithm for determining of movement duration in task space without violating joint angle constraints," in Proceedings of the 2001 IEEE Int. Conference on Robotics and Automation, Seoul, Korea, May 21-26, 2001, pp. 972-978.
8. P.P. Bhangale, V.P. Agrawal, and S.K. Saha, "Attribute based specification, comparison and selection of a robot," Mechanism and Machine Theory, Vol. 39, 2004, pp. 1345-1366.