Robust Feedback Linearization based Control of Robot Manipulator

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ABSTRACT

In this paper a new formulation of a robust trajectory tracking controller for robotic manipulator is proposed. The controller is based on the feedback linearization approach wherein the uncertainties are estimated by employing a novel Uncertainty and Disturbances Estimator making the controller robust. Unlike many formulations employed to robustify the feedback linearization based controller, this approach does not require knowledge of even the bounds on uncertainty. Simulations are carried out by considering significant uncertainty and the results are presented to demonstrate the effectiveness the controller.

Keywords: Feedback Linearization, Nonlinear control, Uncertainty and Disturbance Estimation, Robust control, Robotic control, Trajectory tracking control.

1 Introduction

Geometric control theory based Feedback Linearization [1]-[3] is a well-known strategy for designing controllers for nonlinear systems. Applications of this approach can be found in various disciplines such the aerospace engineering [4] and robotics [5]. One of the advantages offered by feedback linearization is that it provides a systematic framework for designing the controller. In this approach, the basic idea is to obtain a nonlinear controller which seeks to linearize the otherwise nonlinear system. Once the system is linearized, any standard linear technique can be employed for designing the control law. Obviously, the success of this approach hinges on the availability of the accurate description of the model. In presence of model uncertainties, the performance of the feedback linearizing controller cannot be guaranteed to be satisfactory as the nonlinearities do not get cancelled exactly.

Various approaches have been presented in literature to robustify the feedback linearization based controller in presence of uncertainties. In Ref. [5], the problem is addressed by designing the outer loop by using the Second Method of Lyapunov. In [4], robust outer loop design based on the Lyapunov's second method is presented for design of an autopilot of a BTT missiles. In [6], the problem of stabilization of a class of nonlinear systems with timevarying uncertain parameters is presented wherein a Lyapunov function based robust con- troller is incorporated with feedback linearization to guarantee practical stability. In all these approaches the uncertainties are assumed to norm bounded and thus it becomes necessary to know the bounds as assumption of higher bounds result in degradation of performance.

In this paper, the feedback linearizing controller is robustified by estimating the uncertainties using a recently developed Uncertainty and Disturbance Estimator (UDE) technique. [7]. In order to demonstrate the effectiveness of the proposed approach, a trajectory tracking controller is designed for a one rigid link manipulator using feedback linearization and the controller is robustified by using the UDE. This approach does not require knowledge of the bounds of uncertainties.

With this background, the remaining paper is organized as follows. In section 2, the feedback linearization based controller for one rigid link manipulator is formulated. The UDE approach, which has been used to estimate the uncertainties to robustify the FL based controller is presented in Section 3. The simulation results demonstrating the effectiveness of the predictive controller are presented in section 4 and finally Section 5 concludes this work.

2. Formulation of controller

In feedback linearization, the basic idea is to design a nonlinear controller which seeks to linearize the otherwise nonlinear system. Although when applied to a general nonlinear system, this approach necessitates a change of co-ordinates to yield the feedback linearizing controller, for systems such as rigid link manipulators, the controller can be obtained in the original coordinates and the design consists typically of two stepsfirstly construct a nonlinear control law as a so called *inner loop control,* and then design a second stage or *outer loop control* to obtain the desired closed loop performance. In this section, the feedback linearization approach is used to design a controller for a one link rigid robot manipulator. For detailed introduction of this approach, the readers are referred to Refs. [2]-[3].

Consider a single rigid link manipulator the mathematical model of which is

$$I\ddot{q} + B\dot{q} + mgL\sin q = u \tag{1}$$

where q is the link angle, I is the link and actuator inertia, B is the actuator damping, u is the input torque, and m and L are the mass and length of link respectively. The objective is to design a controller such that the link position, $q(t)$ tracks a given reference trajectory, $q_r(t)$.

Re-writing Eq. (1) as

$$\ddot{q} + \frac{B}{I}\dot{q} + \frac{mgL}{I}\sin q = \frac{1}{I}u \tag{2}$$

The idea behind feedback linearization is to seek a nonlinear feedback control law which when substituted in (2), results in a linear closed loop system. The resulting control law is designated as an *inner-loop* control. To achieve this choose the control as

$$u = B\dot{q} + mgL\sin q + I\upsilon \tag{3}$$

where the function $\upsilon(t)$ is an outer-loop control to be designed in the next step. It can be noted that the system (2) under control (3) is linear one. Next the outer-loop control $\upsilon(t)$ is to be designed such that the tracking error dynamics has desired characteristics. In particular, choosing $\upsilon(t)$ as

$$\upsilon = \ddot{q}r - K_1(\dot{q} - \dot{q}_2) - K_2(q - q_r) \tag{4}$$

and substituting (3) and (4) in (2) yields the linearized error dynamics

$$\ddot{e} = K_1\dot{e} + K_2 e = 0 \tag{5}$$

where $e \triangleq q - q_r$ is the tracking error. Clearly, one need to choose K_1 and K_2 to obtain satisfactory tracking performance and stability characteristics. The controller, thus designed will offer satisfactory performance if there are no uncertainties in the system model. However, for practical systems this requirement is seldom satisfied and thus it becomes necessary to address this issue.

Now consider the dynamics given in Eq. (2) and assume that there exists uncertainties in plant parameters i.e. the dynamics takes a form as

$$(I + \Delta I)\ddot{q} + (B + \Delta B)\dot{q} + mgL + \Delta mgL)\sin q = U \tag{6}$$

which can be re-written as

$$\ddot{q} + \frac{B}{I}\dot{q} + \frac{mgL}{I}\sin q = \frac{1}{I}u + \frac{1}{I}d \tag{7}$$

where d represents the total effective uncertainty defined appropriately from Eq. (6). Clearly for this system the controller given by Eq. (3) will not be applicable due to the presence of the uncertainty d. However, if Eq. (3) is modified as

$$u = B\dot{q} + mgL\sin q - d + Iv \tag{8}$$

then one can obtain the same performance as that of a system having no uncertainty. Obviously, this is possible only if the lumped uncertainty given by d can be estimated.

3 Uncertainty and Disturbance Estimator

As stated in the last section, one can robustify the feedback linearization based controller by estimating the uncertainty. To this end, recently developed Uncertainty and Disturbance Estimator (UDE) [7] is used to estimate the uncertainty. The reader is referred to Ref. [7] for a detailed development of the UDE. From Eq. (7), it can be noted that at any time *t*, the uncertainty, *d(t)*, can be obtained as

$$d(t) = I\ddot{q}(t) + B\dot{q}(t) + mgL\sin q(t) - u(t) \tag{9}$$

From (9), it is obvious that the uncertainty d can be estimated using system states and control signal. However, it can not be used in control law directly. Now let $\hat{d}$ be the estimate of d. Then one can approximate the signal d(t) to its estimate $\hat{d}$(t) where

$$\hat{d} = (I\ddot{q} + B\dot{q} + mgL\sin q - u)G_{f(s)} \tag{10}$$

and $G_f\,(s)$ is a first order low pass filter

$$G_f(s) = \frac{1}{1+s\tau} \tag{11}$$

with time constant of τ. The UDE given by Eq. (10) only uses the control signal and the states to observe the uncertainty. Substituting for *d* by its estimate $\hat{d}$ given by Eq. (10) and the outer loop control of Eq. (4) in feedback linearizing controller of Eq. (8) leads to the desired controller which achieves robust performance in presence of uncertainties. The controller thus obtained is

$$u = B\dot{q} + mgL\sin q - \frac{I\dot{q}}{\tau} + \frac{(1+s\tau)}{s\tau}[\ddot{q}_2 I - K_1 I(\dot{q} - \dot{q}_r) - k_2 I(q - q_2)] \tag{12}$$

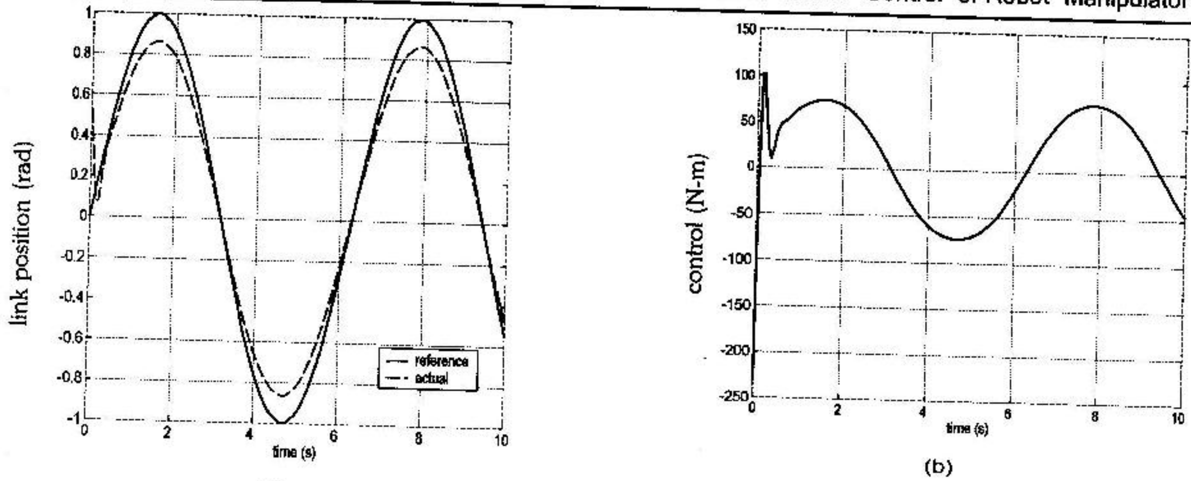
Once can note that the control (12) is formed by the state, low pass filter, feedback gains and the reference state without explicit mention of the uncertainty.

4 Numerical Simulations

To demonstrate the effectiveness of the present formulation, simulations are carried out by taking *B=1, mgL = 10, I = 2* as nominal values. The value of time constant of the filter of Eq. (11) is taken as 0.005. The values of gains $K_1 = 16$ and $K_2 = 256$ are chosen to satisfy the requirement of settling time of approximately 0.5 sec with closed loop damping of 0.5. The desired trajectory, $q_r = \sin(t)$ has been chosen. Now mismatch in initial condition of link position is introduced by taking $q(0) = 30°$ and the value of load i.e. *mgL* is taken as 100 instead of its nominal value of 10. With this uncertainty and initial mismatch, simulations are carried out by using the feedback linearization based controller of Eq. (3) and the robustified controller given by Eq. (12) and the results are presented in Figs.1 and 2 respectively. It can be observed from the Fig. 1(a) and Fig. 2(a) that the UDE based robust controller has enhanced the tracking performance in presence of significant uncertainty. The corresponding control histories are given in Figs. 1(b) and 2(b).

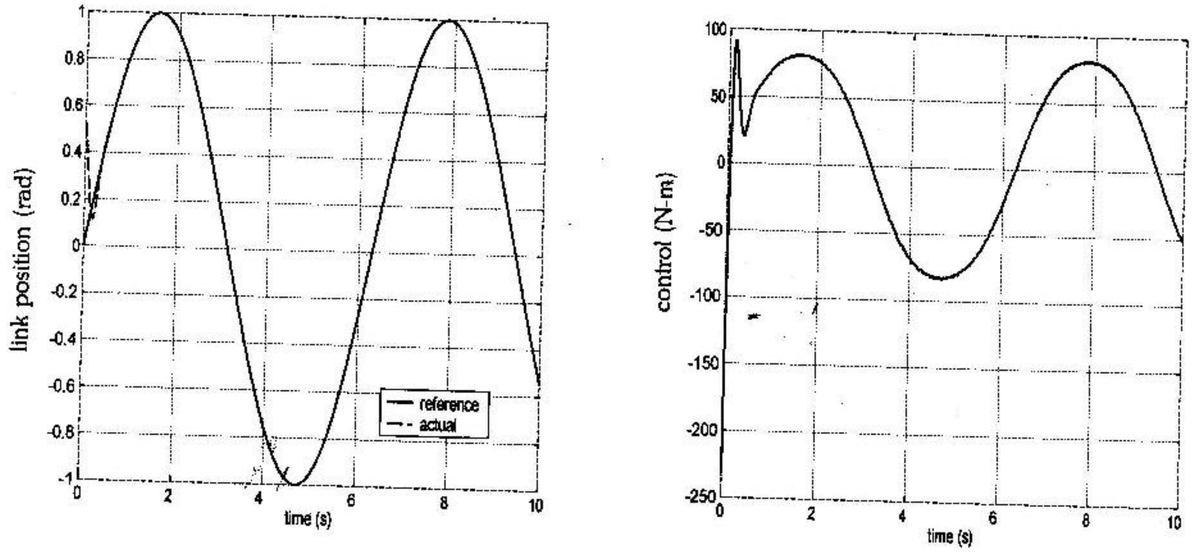
5. Conclusion

In this paper, uncertainty and disturbance estimation approach is used to robustify the feedback linearization based controller as applied to trajectory tracking problem of rigid link robot manipulator. Simulations are carried out and the results are presented. The results demonstrates the effectiveness of the formulation in presence of significant uncertainty.



(a) (b)

Figure 1 : Tracking performance without UDE



(a) (b)

Figure 2 : Tracking performance with UDE

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