Closure Grasps using Qualitative Kinematics

S. M. Hazarika

Department of Computer Science & Information Technology
Tezpur University, Tezpur
India
email:smh@tezu.ernet.in

Abstract

This paper extend results from *Qualitative Kinematics to* formulate a grasp synthesis procedure. The grasps so obtained are shown to satisfy the efficiency criteria given by *Quantitative Steinitz's Theorem.*

1 Introduction

Grasping an object consists of finding a set of *fingers* whose contact with the object prevents its motion. Grasping is based on closure properties - form closure and force closure. A rigid body is said to be in *form closure* if a set of contacts along its boundary constraints all finite and infinitesimal motions of the body [11, 3]. *Force closure* is related with the capability of the *fingers being* considered to apply forces through contacts [2]. In this paper, grasp refers to a form closure grasp.

A *qualitative* formalism is used to arrive at a grasp. The motivation for a qualitative approach to grasp synthesis is the observation, that everyday interaction with the physical world is driven through *qualitative* abstractions, rather than complete quantitative knowledge a priori. Moreover, a *qualitative approach* arrives at a so lution through a simpler process than classical kinematic analysis. However, it retains important distinctions of kinematic behaviour of objects (without invoking the myriad equations in- cluding differential equations) [12].

A procedure is developed to compute location configuration of point constraints for immobilization of an object in 2D, leading to a qualitative approach to grasp synthesis. The grasps so obtained satisfies the efficiency criteria of closure grasps as specified by the quantitative Steinitz's theorem [9].

2 Qualitative Kinematics

Qualitative Kinematics is concerned with the mechanical interactions of objects described through qualitative abstractions [8, 4]. The goal is to develop methods for constructing qualitative descriptions of freedom of motion sufficient to understand a mechanism using the principles of *qualitative physics* [5, 6, 12]. The paper exploit results from reasoning about mechanical interaction of objects [10] within *qualitative kinematics* - the sub-field of qualitative physics concerned with spatial reasoning required by commonsense physics. A complete discussion of qualitative kinematics here is neither intended nor feasible. For an in-depth discussion see [6].

2.1 Mechanical constraint

Qualitative mechanics develops a commonsense theory of mechanical analysis sufficient to describe the behaviour of rigid body devices [7, 10]. In 2D there are three degrees of freedom: two translational and one rotational motion about an axis perpendicular to the plane. The constraints which may be imposed when two objects are in contact was first explored by [10]. Figure 1 shows the constraints imposed by placement of a point contact P_i on the boundary of an object O.

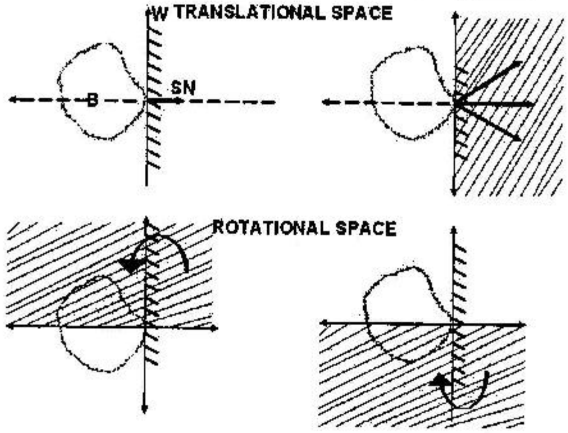

Figure 1: Constraints imposed by surface contac remaining translational and rotational motion is shown (From [10]).

According to Nielsen [10], if an obstacle is *sufficiently* constrained it will prevent the following motions of an object in contact:

a. Translational motion into the open half-plane centred on the objects surface normal at the point of contact.

b. Rotational motion clockwise about any axis which lies in the open halfplane centred 90 degree clockwise from the objects surface normal at the point of contact, or

c. Rotational motion counter clockwise about an axis which lies in the open halfplane centred 90 degree counter clockwise from the object surface normal at the point of contact.

In section 3.1, the above result from quali-tative kinematics is exploited to define *motion spaces*. Based on the interaction of motion spaces from more than one point constraints, m - finger grasps for a 2D object is synthesised.

3 Qualitative Grasp Synthesis

The general case considered for any grasping procedure is described as follows:

The procedure under assumption finds grasp for a rigid object O. The fingers will be placed at points on the boundary of O, which is denoted by δO Only the points of contact of this hand with O, are considered and issues such as motion planning and accessibility are ignored.

The following definitions are introduced.

Definition 1 *Smooth Rigid Body: A smooth rigid body* O *is a closed compact subset of the* Euclidean 2-space. *Furthermore,* O *has a piece- wise smooth boundary* δO.

Definition 2 *Point Contact: For each finger-contact on the body a nominal point of contact,* $P_i \in \delta O$, *denotes a contact which is*

a. non-singular i.e., δO *has a unique normal at each such point*

b. frictionless[1] .

With object O and point contact P_i defined above, a grasp is defined. A grasp consist of m - points on the boundary of the body to be grasped.

Definition 3 *Grasp: An m-finger grasp* $\mathcal{G}$ *of an object* O, *is a set of m-points, where* $\mathcal{G} \subset \delta O$ *ensures*

[1] *Frictionless point contacts implies that forces canonly be applied along the normal at the point of contact, directed inward into the object*

positive grip (i.e., $\mathcal{G}$ is the set of point con- tacts $\{P_1, P_2 . . . . P_m\}$). Appropriate force on the point contacts contain all finite and infinitesimal motions of O keeping it in equilibrium.

3.1 Qualitative Analysis

From Nielsen's analysis of mechanical constraint[10], the following *motion spaces* for translational as well as rotational motion of an object O with respect to a point contact P_i is defined.

Definition 4 *Translational Space: Given an object O and a point contact P_i, translational space $\Im_i$ is a subset of the Euclidean 2-space about which O can have translational motion.*

Definition 5 *Rotational Space: Given an object O and a point contact P_i, rotational space $W_i \subset$ Euclidean 2-space, such that O can have rotational motion about any axis which lies in W_i.*

Definition 6 *Positive Rotational Space: Given an object O and a point contact P_i, positive rotational space $W^+_i \subseteq W$, such that O can have rotational motion clockwise about an axis which lies in the open halfplane centred 90 degree clock- wise from the normal at point of contact.*

Definition 7 *Negative Rotational Space: Given an object O and a point contact P_i, negative rotational space $W^-_i \subseteq W$, such that O can have rotational motion counter clockwise about an axis which lies in the open halfplane centred 90 degree counter clockwise from the normal at point of contact.*

A constraint is applied along the boundary of an object. Based on the point contact P_i on the object, the space around which the body can move is partitioned in two halfspace. The two halfspace are discrete spaces, since their properties are different. Considering the direction of application of constraint as viewing direction the body has clockwise rotation in right-hand halfplane and counterclockwise rotation in left- hand halfplane. The clockwise halfplane is W^+ and counterclockwise halfplane is W^-. Whenever due to application of constraints,there is an overlap of two discrete halfplane, it creates a *Null* space. In the *Null* space the body ceases to have certain degrees of freedom (which it possessed earlier). Figure 2 illustrates this idea.

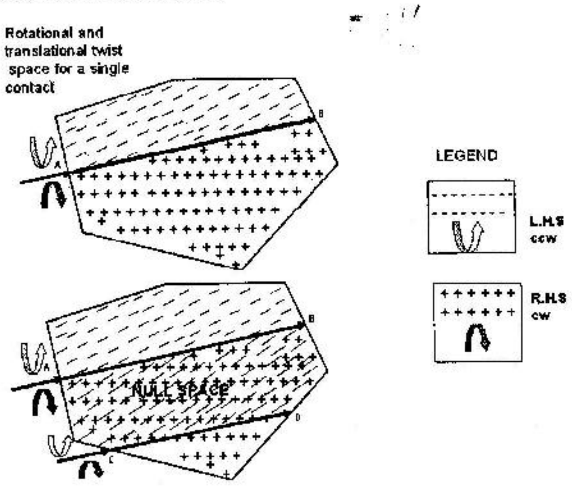

Figure 2: Constraints leading to a *Null* space.

In Figure 2 after constraint is placed at A, the body has W^+ in right halfplane and W^- in left halfplane. After the second point contact is applied at C, the body ceases to have rotational space in the region ABDC. The set of constraints are to be generated in such a way that each pair of halfspace created by the consecutive constraints cancels each other to the maximum extent and finally after the specified number of constraints are placed there should not be any translational or rotational space left. This configuration of the body is said to achieve the state of immobility.

The translational space $\Im_i$ and rotationalspaces W_i^- as well as W_i^+ is because of a single point contact P_i. The *motion spaces* because of each contact on an object intersects. The ability of an object (constrained by more than one point contact) to have any finite and/or infinitesimal rotational or translational motion depends on the intersection of the individual *motion spaces. A zone of freedom Z* is introduced.

Definition 8 *Zone of Freedom: For a body O zone of freedom Z is the resultant rotational and translational space the body has after constraints are applied at set of point contacts* $\{P_1, P_2 \ldots P_m\}$.

Given the above definition of *zone of freedom Z , it is now possible to define a qualitative grasp.*

Definition 9 *Qualitative Grasp: A m-fingered qualitative grasp Q of an object O is a set of m-points where*

a. $Q \subset \delta O$

b. each of the m-points is a point contact P_i and

c. *Z resulting from the m point contacts is NULL*

4 Qualitative Grasp Algorithm

4.1 Qualitative formulation

First partition the region in two discrete halves by a constraint along the line AB, positive half representing W_i^+ and negative half representing W_i^-

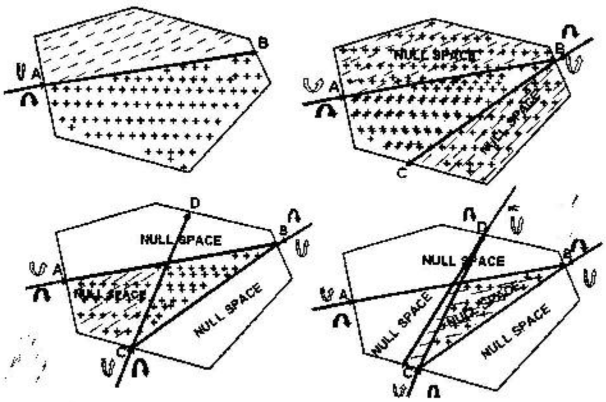

Figure 3: NULL Space based on successive partitions.

Next partition line BC has two possibilities: it is either on LHS or on RHS of the previous line AB. If it falls on LHS of AB than negative area remains else positive area remains (as shown in Figure 3).

If W^+ remains, the next partition line CD may fall either on LHS or on RHS of BC. If CD falls on RHS of BC the whole area turns to be NULL. But if it falls on LHS of BC, then W^+ as shown in Figure 3 remains.

Similarly, if W^- remains the next line CD may fall either on LHS or on RHS side of BC. If CD falls on LHS of BC the whole area turns to be NULL but if it falls on RHS of BC then W^- remains.

Note that the translational space $\mathfrak{T}$ is dependent on the rotational space W^2. Therefore bringing the intersection of W_i s, because of a set of point contacts P_i s to NULL would lead to $\mathfrak{T}$ being NULL for the given set of point contacts.

Generalized qualitative formula

The generalized qualitative formula for determining NULL space i.e. $Z = 0$ starts for $n \geq 3$ where n is the number of partition. There are two cases:

[2]Translation in a plane can be seen as rotation about an axis perpendicular to the plane through a point at infinity.

1. **If positive area remain and**

- if n^{th} partition boundary falls in negative section with respect to(n-1)th partition boundary then total NULL space is created or $Z = 0$ condition is achieved.
- if n^{th} partition boundary falls in positive section with respect to (n-1)th partition then positive area remains.

2. if negative area remain and

- if n^{th} partition boundary falls in positive tive section with respect to (n-1)th partition then total NULL space is created or $Z = 0$ condition is achieved
- if n^{th} partition boundary falls in negative section with respect to (n-1)th partition then negative area remains

4.2 Procedure: *QualGrasp*

1. Create the set S of non-singular points along the boundary of the object δO.
2. Repeat until S is empty.

   a. Select any point $p_i \in S$.

   i. Repeat until $Z = 0$ or m exceeds specified number of points.

   ii. Apply point contact P_i at p_i

   iii. Calculate all intersecting edges of the object with the line of action of constraint. Take the maximum distance of the intersecting points.

   iv. Based on the partition of the initial point and the final point selected on the object boundary, two discrete spaces are identified.

   v. Compute Z.

   b. Give a grasp-id to the m -points. Delete p_i from S where the current procedure started.

Theorem 1 *Any grasp $\mathcal{G}$ synthesized by procedure QualGrasp (for any object O) is an qualitative grasp.*

Proof of above theorem is straightforward based on Definition 9 and Procedure QualGrasp.

5 Steinitz's Theorem

Barany, Katchalski and Pach [1] showed that the common quantitative version of Steinitz's Theorem holds. The following theorem from [1] is referred to as the Quantitative Steinitz's Theorem.

Theorem 2 *For any positive dimension d, there is a constant $r = r(d) > d^{-2d}$ such that given any set $S \subset E^d$, where E is the Euclidean space of points in d-space whose convex hull contains the unit ball centered at the origin o, there is a subset $X \subset S$ with at most 2d points whose convex hull contains a ball centered at o with radius r_d.*

5.1 Closure property of Q

A desirable notion of grasping is that of a closure grasp. The Quantitative Steinitz's Theorem gives a measure of efficiency of such grasps. The quantity $r_d(m)$, where r_d is the largest residual radius with at most m -points, gives this efficiency directly [9].

Theorem 3 is the Quantitative Steinitz's Theorem in 2 dimensions from [9]. Theorem 3 gives bounds for r_2 for m-points.

Theorem 3 *In 2 dimensions, for all $m > 4$, we* *have* $\frac{3\pi^2}{2(m+1)^2} < 1 - r_2(m) < \frac{2\pi^2}{m^2}$

The above result is used here in determining the efficiency of the qualitative grasps *Q* obtained on any object *O* through QualGrasp .

Example 1 Consider an object *O* = {(15, 1), (4, 1), (6, 3), (4, 4), (9, 7), }. Co-ordinates of the centre of unit ball is (9.5, 2.5) and radius 5.7.

5-fingered planar grasp Q={(5, 3.5), (5.78, 5.07), (8.23, 1), (8.23, 6.54), (11.51, 1)} is obtained using procedure QualGrasp . The centre of unit ball of the grasp point is (8.54, 2.96) and radius 3.6. For m = 5 in 2D, above values satisfy Theorem 3 as 0.411 < 0.64 < 0.79.

Q is an efficient closure grasp as it satisfies Qualitative Steinitz's Theorem.

Example 2 Consider an object *O* = {(9, 1), (4, 1), (6, 3), (4, 4), (9, 7), (12, 6), (16, 7), (18, 4), (13, 3), (15, 1)}.Co-ordinates of the centre of unit ball is (11, 2.5) and radius 7.1.

5-fingered planar grasp Q={(7.4, 1), (7.4, 6.04), (10.42, 1), (10.42, 6.52), (8.58, 1)} is obtained using procedure *QualGrasp*. The centre of unit ball of the grasp point is (8.5, 3.5) and radius 2.9. For m = 5 in 2D, above values satisfy Theorem 3 as 0.411 < 0.71 < 0.79.

Q is an efficient closure grasp as it satisfies Qualitative Steinitz's Theorem.

6 Final Comments

In this paper, a qualitative approach to synthesis of planar grasps based on qualitative kinematics is proposed. A simple grasping algorithm QualGrasp based on qualitative analysis of mechanical constraint for two dimensional objects and complete restrain of the object under point contacts have been developed. Using results from Quantitative Steinitz's Theorem, it has been shown that the grasps obtained are efficient closure grasps.

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