Structural Synthesis of Kinematic Chains -Application of CLAM along with Secondary Hamming String

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ABSTRACT

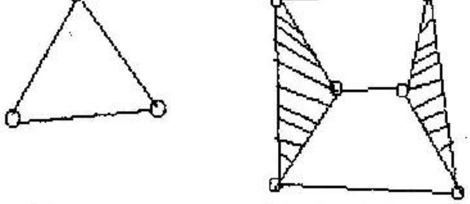
Number synthesis, enumerations, identification of degenerates, isomorphism and classification is the areas of studies under structural synthesis. Institutive approach, Franke's notation, Assur technique, Theory of Graph, Transformation of multi-jointed chains and other mathematical methods are used particularly for enumeration and isomorphism problem. Combination of Contracted link adjacency matrix (CLAM) as suggested by Hwang and Hwang for enumerations and Hamming number technique as suggested by A.C.Rao for detection isomorphs was used . All the 230 combinations of 10-bar chain in the form of adjacency matrix were obtained. For this purpose suitable algorithm was developed to convert CLAM to adjacency matrix, which subsequently converted to Primary Hamming matrix. Initially primary Hamming strings were used for testing isomorphism but as expected we obtained 228 chains as against 230. Primary Hamming string could not distinguish between chain No 1 and 40 as well as chain No 23 and 50. Hence secondary Hamming string was used. 3-link and 5-link degenerates were identified suitably. Important feature of the work has been application of contracted link adjacency and Hamming number technique together and conversion of contracted link adjacency matrix to adjacency matrix.

INTRODUCTION

Reuleax was first to make systematic attempt for synthesis of mechanisms though it was focused on type synthesis [1, 2]. R.S Hartenbrrg and J. Denvait had given recognition to synthesis as independent area of study by publishing their book Kinematic Synthesis of Linkages [3]. Many Kinematcians had given glory to the Kinematic Synthesis by different approaches to the synthesis problem. Broadly these approaches and contributions by researchers; though interrelated are caegarised as follows. (1) Intuitive Approach: M. Grubler, K. Hain, F.R.E. Crosslley [4-6]. (2) Assure Technique: L.V. Assure, N.I.Monolescu [7,18]. (3) 14]. (5) Transformation of Multi-jointed Chains: N.I.Monolescu, T.S. Mruthynjaya [15-18].(6) Other Mathematical: W.M.Hwang , Y. W. Hwang ,V.P.Agrawal, A.C.Rao, D.Varada Raju, C. N. Rao[6,19-23]. As mentioned earlier this is only a broad spectrum from the literature authors have referred [7].

MATHEMATICAL REPRESENTATION

Redundant Chain: If there is no relative motion between the links then that chain is referred as redundant chain. Formation of redundant chain may occur because of triangular loop or short circuited E-quartet as shown in Fig 1 (a) and (b) respectively.



(a) Triangular loop (b) Short circuited E-Quartet

Figure 1: Redundant Chain

Degenerate Mechanism: All the chains, which consist of formation of some rigid part due to presence of triangular loop or E-quartet, are referred as Degenerate mechanisms/chains [4].

Adjacency Matrix: The adjacency matrix of a graph G with n-vertices and no parallel edges is an $n \times n$ symmetric binary matrix $A= [a_{ij}]$ defined such that

$a_{ij} = 1$; if there is an edge between i^{th} and j^{th} vertices.

$=0$; if there is no edge between them [24].

Adjacency matrices are equivalent if they differ only by permutation of rows and column.

Matrix 'A' can be considered as the link -link incidence matrix of kinematic of 'n' number of links [15].

Hamming Number: The Hamming Number for any two codes each with 'n' digits may be defined as the total number of bits in which the two codes differ with each other [20]. Here we refer codes as combination of '1' and '0' obtained for ith and jth rows in Adjacency Matrix 'A'.

Applying above definition to rows 'i' and 'j' of 'A' we have .

$$h_{ij} = \sum_{k=1}^{n} S_k$$

Where $S_k = 0$ if $a_{ik} = a_{jk}$

Isomorphic Chains: Two kinematic chains are isomorphic if their adjacency matrices are equivalent. Fig- 2 (a) and (b) shows two isomorphic forms of Watt's chain. Fig-2 (c) shows graphical representation of Watt's chain and Fig-2(d) shows adjacency matrix 'A' for one form of Watt's chain [11].

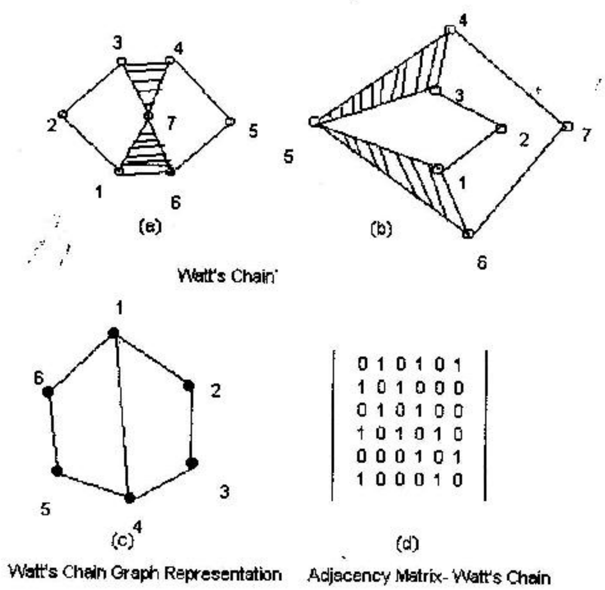


Figure 2: Watt's Chain

The Hamming Number between any two rows of adjacency matrix of size 'n' can be any positive integer from 'n' to 0. It will be n if all the elements are different and will be zero if the two rows are exactly identical. Hamming matrix is square, symmetric and contains zeros all along the leading diagonal. However unlike adjacency matrix it contains digits which could be larger than unity as shown in matrix 'H' ahead.

Link Hamming Number for any link 'i' is the sum of all the elements in the ith row of the Hamming matrix.

Chain Hamming Number for any chain is the sum of all the link Hamming numbers of that chain

Link Hamming String for any link 'i' is the string obtained by concatenating (a) the link Hamming **Number of 'i with** (b) the frequency of occurance, of all the integers from n downto 0, in the Hamming **Numbers** of that row 'i'.

Chain Hamming String is defined as concatenation of the (i) Chain. Hamming Number and (ii) link Hamming strings, these strings placed in decreasing order of magnitude.

Secondary Hamming Matrix can be constructed, working in the similar fashion on primary Hamming Matrix.

Link Hamming numbers ↓

$$H = \left|\begin{matrix} 0 & 5 & 1 & 6 & 1 & 5 \\ 5 & 0 & 4 & 1 & 4 & 2 \\ 1 & 4 & 0 & 5 & 2 & 4 \\ 6 & 1 & 5 & 0 & 5 & 1 \\ 1 & 4 & 2 & 5 & 0 & 4 \\ 5 & 2 & 4 & 1 & 4 & 0 \end{matrix}\right| \begin{matrix} 18 \\ 16 \\ 16 \\ 18 \\ 16 \\ 16 \end{matrix}\right|$$

Total chain Hamming number 100

Therefore chain Hamming string :

100, 18, 1200021, 18, 1200021, 16, 0120111, 16, 0120111, 160120111, 16, 0120111.

CLAM: (Contracted Link Adjacency Matrix) A Kinematic chain is composed of two kinds of links **viz.** polygonal links and binary links. Each polygonal link and also each string of binary link, called contracted links, are regarded as a unit. Thus, the CLAM of a Kinematic chain with n units is expressed as $n \times n$ matrix. Each principal diagonal element of the CALM, ei i (i =1 to n) is defined such that eii = f (if unit 'I' is a polygonal link with 'f' joints) and ei i = -m (if unit is contracted link with 'm' binary links). In order to construct the CLAM the largest value of the diagonal element is placed as the top element of the first (left) columns. The other numbers of the diagonal elements is placed as the bottom element of the last (right) column [6].

The off-diagonal elements e_{ij} of the CLAM is defined such that e_{ij} = t if unit- i and unit-j are connected by't' joints and e_{ij}=0 otherwise. CLAM for Watt's chain in Figure 2 is shown below.

$$CLAM = \left|\begin{matrix} 3 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & -2 & 0 \\ 1 & 1 & 0 & -2 \end{matrix}\right|$$

SELECTION OF METHOD

To obtain the number of Kinematic chains up to 10 links with single degree of freedom for enumerations three options were studied

1) Contracted Mapping [11]

2) Transformation of Binary Chains [18]

3) Contracted Link Adjacency Matrix Method [6]

After studying all these methods, the contracted Link Adjacency Matrix Method discussed by Hwang and Hwang [6] was selected as it was observed there was possibility to develop efficient algorithm with simplicity to obtain desired results.

Detection of 3-link and 5-link degenerate was done by development of suitable algorithm by referring the properties of three link redundant chains and five link E-quartet chains.

As characteristic polynomial method fails for detecting the isomorphs for 10-link chains [17] other options were studied. Formation of some of the isomorphs was avoided as discussed by Hwang and Hwang [6] and remaining isomorphs were eliminated by comparing Secondary Hamming String as discussed by Rao and Varda Raju [20] because of simplicity of this method it was selected.

However, in first phase of program development Primary Hamming string was tried to detect isomorphism; but only 228 chains were obtained as against 230 Nos. required for 10-link single degree of freedom. Primary Hamming string could not distinguish between chain No 1 and 40 as well as chain No 23 and 50 from Crossley's collection.[11] Hence Secondary Hamming String Method was used. For that purpose CLAM was converted to Adjacency Matrix, Primary Hamming Matrix and lastly to Secondary Hamming Matrix.

ALGORITHM

Major steps taken while developing C-program are as follows:

1. Read the input links and degrees of freedom
2. Link assortments were obtained using Crossley's Algorithm [4, 5]
3. Contracted Link Assortments were obtained by development of algorithm parallel to Crossley's algorithm.
4. Selected one combination of Link Assortment and Contracted Link Assortment. Diagonal elements of CLAM were obtained.
5. Off diagonal elements were obtained for CLAM with due care to avoid some isomorphic forms [6] Thus CLAM is obtained.
6. This matrix was tested for degenerate form of 3 links and 5 links and also tested for closed form.
7. If it is not degenerate or open loop switched to next step otherwise to step (4) to obtained next possible CLAM.
8. CLAM is converted to Adjacency Matrix by developing suitable algorithm.
9. Adjacency Matrix converted to Primary Hamming Matrix [20]
10. Primary Hamming Matrix converted to Secondary Hamming Matrix [20]
11. Secondary Hamming String is obtained. First string obtained stored directly else string is compared with the stored strings if the newly obtained string is different then it is stored. If string is stored output i.e. corresponding CLAM, Adjacency Matrix and Hamming String is printed.
12. Switched back to step (4) till all possible combinations of CLAM for selected Link Assortment and Contracted Link Assortment; are obtained.
13. Switched back to step (3) till all possible combinations of Link assortments and contracted link assortments are obtained.

Due care was taken for few things such as to provide bypass if number of link are less or equal to 4 or greater than 10 or odd or degree of freedom is not equal to 1. Similarly due care was taken to test for primary Hamming Strings only for number of links only up to 8.

Steps carried out for conversions of CLAM to Adjacency Matrix were as follows:

1. Dimension of Adjacency Matrix is (No. of Links × No. of Links).
2. Initialized all elements Adjacency Matrix to zero.

3. For diagonal element of CLAM e_{ii} = -1 all off diagonal elements i[th] row to i[th] column are same as that of CLAM.
4. For other case i.e. e_{ii} = -2 corresponding to which rows if e_{ki} = 0 then corresponding elements of Adjacency Matrix ak i and ak i+1 are set to zero and if ek i = 1 then ak i is set as 1 and ak i+1 = 0. Now for any element e_{ji} =1 then a_{ji} = 0 and a_{ji}+1 = 1.
5. If now $e_{i+1,i+1}$ = -2 then next corresponding rows and columns are i+2 and i+3 care for step (4) is taken.
6. Care for Adjacency Matrix is symmetric about zeros as diagonal elements are taken.

Conditions imposed to detect degenerate of 3 links and 5 links

a) 3-link-degenerate

If $e_{ij} = e_{jk} = e_{ik} = 1$ for upper left sub matrix off diagonal elements of CLAM i.e. for which e_{ii} = -1 is satisfied it is noted as degenerate.

b) 5-link-degenerate

If $e_{il} = e_{im} = e_{in} = e_{jl} = e_{jm} = e_{jn} = 1$ for upper sub matrix off diagonal of CLAM then it is noted as degenerate.

'C' program is developed using the approach and algorithm discussed [25, 26]. Microsoft VC++ environment is used for the same.

RESULTS:

Results of the output for the 4 links, 6 links, 8 links and 10 links were compared with the established results. Number of chain obtained during the test runs are shown in Table -1

Result obtained for only first and last chain in matrix form for 10-bar mechanism is shown below. CLAM matrix, Adjacency Matrix, Secondary Hamming String is obtained for all the 230 chains foe 10-bar chain.

Table -1

Results Obtained for Test Runs

No. of links	No. of single Degree of Freedom Chains obtained
4	1
6	2
8	16
10	230

-------- Output -------------

Matrix no: 0

-- CLAM Matrix --

```
3 1 1 1 0 0 0 0
1 3 0 0 1 1 0 0
1 0 3 0 1 0 1 0
1 0 0 3 0 1 1 0
0 1 1 0 3 0 0 1
0 1 0 1 0 3 0 1
0 0 1 1 0 0 -2 0
0 0 0 0 1 1 0 -2
```

-- Adjacency matrix --

0 1 1 1 0 0 0 0 0 0
1 0 0 0 1 1 0 0 0 0
1 0 0 0 1 0 1 0 0 0
1 0 0 0 0 1 0 1 0 0
0 1 1 0 0 0 0 0 1 0
0 1 0 1 0 0 0 0 0 1
0 0 1 0 0 0 0 1 0 0
0 0 0 1 0 0 1 0 0 0
0 0 0 0 1 0 0 0 0 1
0 0 0 0 0 1 0 0 1 0

.... Hamming string...

5624=572(=4*76+2*74+1*56+1*40+1*24)+572(=4*76+2*74+1*56+1*40+1*24)+572(=4*76+2*74+1*56+1*40+1*24)+572(=4*76+2*74+1*56+1*40+1*24)+560(=1*80+4*76+1*62+1*48+1*40+1*26)+560(=1*80+4*76+1*62+1*48+1*40+1*26)+560(=1*80+4*76+1*62+1*48+1*40+1*26)+560(=1*80+4*76+1*62+1*48+1*40+1*26)+548(=1*76+4*74+2*62+2*26)+548(=1*76+4*74+2*62+2*26)

-------- Output -------------

Matrix no: 229

-- CLAM Matrix --

5 0 1 1 1 1 1
0 5 1 1 1 1 1
1 1 -1 0 0 0 0
1 1 0 -1 0 0 0
1 1 0 0 -2 0 0
1 1 0 0 0 -2 0
1 1 0 0 0 0 -2

-- Adjacency matrix -

0 0 1 1 1 0 1 0 1 0
0 0 1 1 0 1 0 1 0 1
1 1 0 0 0 0 0 0 0 0
1 1 0 0 0 0 0 0 0 0
1 0 0 0 0 1 0 0 0 0
0 1 0 0 1 0 0 0 0 0
1 0 0 0 0 0 0 1 0 0
0 1 0 0 0 0 1 0 0 0
1 0 0 0 0 0 0 0 0 1
0 1 0 0 0 0 0 0 1 0

.... **Hamming string...**

4856=776(=6*88+1*84+2*82)+776(=6*88+1*84+2*82)+424(=2*88+3*56+2*36+2*4)
+424(=2*88+3*56+2*36+2*4)+424(=2*88+3*56+2*36+2*4)+424(=2*88+3*56+2*3
6+2*4)+424(=2*88+3*56+2*36+2*4)+424(=2*88+3*56+2*36+2*4)+380(=2*82+6*3
6)+380(=2*82+6*36)

No of output matrices in this step: **1**

Total output matrices so far: 230

CONCLUSIONS:

Combination of CLAM and Hamming Number Technique is used together. CLAM is converted to Adjacency Matrix and results obtained in the form of Adjacency Matrix which is suitable form for obtaining graphical representation. Two pairs of chains for 10-links, which fails to Primary Hamming String Test, were obtained. Primary Hamming string, could not distinguish between chain No 1 and 40 as well as chain No 23 and 50 from the collection in [5]. It was also observed that these chains are having crossed links when compared with each other. Detection of degenerate was also important part of this work.

FUTURE SCOPE

With suitable modifications scope of present work can be extended to chains with number of links greater than ten or degree of freedom more than one. Motives of all these studies can be directed towards generation of expert system particularly for selection of robot grippers.

ACKNOWLEDGMENTS

Authors are grateful to all those Kinematcians and researchers from whose contribution to the subject they were inspired. Authors are also thankful to Prof. Pratap Borade , Dr. W.Z. Gandhare ,Dr B.M.Dabde for encouragement and also to Mr. Shirish Gosavi and Mr. Mandar Kshirsagar for valuable suggestions and help extended.

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