Finite Element Analysis of Stiffened Rotating Disc

B.V.V.S.S. Ramanjaneyulu[1], **Anirvan Dasgupta**[2]
Department of Mechnical Enginrreing Indian Institute of Technology, Kharagpur-721 302

ABSTRACT

This paper addresses the effect of taper stiffeners and material properties on critical speeds of very thin spinning disc. Here the stiffener thickness is varied from inner radius to outer radius. In this paper, the circular disc material is taken to be a polymer, while the stiffener materials are aluminum and steel. The number of stiffener and geometry of the disc is also varied. The disc with stiffener is modeled using finite element method (ANSYS6.0) with clamped inner boundary and free outer boundary. From the analysis we found that the critical speeds are improving with increase in thickness and number of stiffeners. The steel stiffeners gives better results when compared to aluminum stiffeners.

Keywords: Natural Frequency, Critical Speeds, Finite element analysis;

NOMENCLATURE

σ_r	**Radial stress.**	σ_θ	Circumferential stress.
$\tau_{r\theta}$	**Shear stress.**	ω	Angular velocity.
ρ_1	**Mass density of disc**	ρ_2	Mass density of stiffener.
u	Radial component of displacement.	v	Hoop component of displacement.
w	Axial component of displacement.	φ	Stress function.
Δ	Laplace operator.	ν_1	Poisson's ratio of stiffener material.
ν_2	Poisson's ratio of disc material.	f(r)	Function of position vector.
$\tau(t)$	Time function.	h_1	Thickness of disc.
h_2	Thickness of stiffener.	r_i	Inner radius of disc.
r_o	Outer radius of disc.		

Introduction

Spinning circular discs at high speed are found in various applications such as huge rotors of steam and gas turbines, gyroscopes, computer hard disks, circular saw blades and spinning membrane for spacecraft radar etc. Such applications may have single or multiple discs attached to a spindle, which is mounted on ball bearing or hydrodynamic bearing. The failures of turbine disc have long been attributed to transverse vibration. This kind of periodic motions of rotating discs has been investigated widely, only, however, in its linear form. Vibration in such system is detrimental and is the root cause of unexpected and undesirable behavior and deterioration of performance. High speed spinning disc can have finite amplitude of deflection and thus the non-linear effect can play a significant role in determining the vibrations in rotating disc.

One of the critical and high precision applications of high speed spinning disc is computer hard disc drive (HDD). For example for data storage, the radial track destiny at present in about 20000 tracks per inch (tpi) and operating speed of about 10000 revolution per minute (rpm)

In order to increase the storage capacity and the access speed, it is demanded that the track density be increased to 25000-tpi and operating speed higher than 15000 rpm. It can well be imagined that, at such speeds and data densities, reading accuracy, and hence its performance will be decided by the vibration of discs. This project is directed towards understanding the problem of vibration in high speed spinning discs for such critical application with the aim of achieving high speed high performance disc drives.

The causes of vibration in high speed spinning discs are many. Primary among them are disc imperfection (such as unbalanced forces, improper mountings etc.), induced vibrations due to defects in the spindle-bearing system, and external forces. Further at the high speeds, the effect of coupling of the disc with the surrounding fluid may provide significant transverse forcing.

The vibration modes of stationary disc are represented by a pair $\omega(m, n)$, where m and n are the number of nodal circles and number of nodal diameters respectively. The corresponding frequency is denoted by ω_{mn}. The effect of rotation of the disc is to cause the standing waves associated with the ordinary natural modes of oscillation of the disc to be split into component traveling wave parts. The frequency of the backward traveling wave decreases, while the frequency of the forward traveling wave increases. This is termed as frequency splitting. At certain spin speed termed as critical speed, the frequency of the backward traveling wave becomes zero. Any non-zero load applied to the disc at the critical speed produces unbounded transverse oscillation amplitudes (in the linear theory) in the absence of any energy dissipation mechanism. Therefore the operating speeds of the disc in upper-bounded by the critical speed of the disc. Hence, it is evident that, to be able to increase the spin speed without adversely affecting performance, the critical speed has to be raised higher that the desired operating speeds of the disc.

The past work was carried out to increase the critical speed of the disc for high-speed applications. The methods to increase the critical speed of the disc are

1. Using taper disc.
2. Using internal channels on the disc.

In present work we propose an innovative idea of increasing the critical speed by introducing the stiffener on the disc with radially decreasing from inner radius to outer radius.

Objective

The objectives of present work are:

1) To observe the improvement in critical speeds of the of disc when stiffener of varying thickness form inside to outside is mounted to the disk
2) To study the effect of increasing of number of stiffeners on critical speed.
3) To study the effect change material of stiffeners on critical speed.

Geometry of Disc with Stiffeners

Let us consider a flat disc, circular, elastic disc of inner and outer diameters equal to 20mm and 51mm respectively, with an external stiffener of radially varying thickness as shown in Fig. 2. The thickness of the disc is assumed to be small and symmetric about a neutral plane, which allows one to treat it as a plane stress problem. First, the equations of motion of a disc with varying thickness are derived, from which the uniform thickness case will follow. All the following derivations are performed in the inertial space.

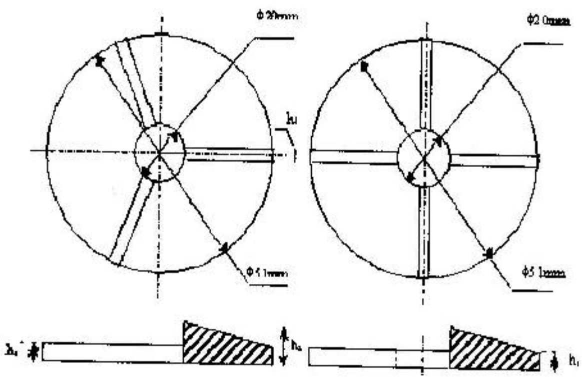


Fig 1: Metal stiffener representation on the polymer disc

Material Properties

		Polymer	Aluminum	Steel
Young's Modulus	(GPa)	40	80	190
Poisson Ratio		0.25	0.33	0.3
Density	Kg/m3	2700	3000	7800

Equation of Motion

Assume a cylindrical polar coordinate system at the center of the disc with the r and θ axes lying on the neutral plane. The strains at any level z from the neural plane are obtained by substituting the classical plate deformation kinematic relations in the non-linear strain displacement relations as (see [18])

$$\varepsilon_r = \frac{\partial u}{\partial r} - z\frac{\partial^2 w}{\partial r^2} + \frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2 \tag{1}$$

$$\varepsilon_\theta = \frac{u}{r} - \frac{z}{r}\frac{\partial w}{\partial r} + \frac{1}{r}\frac{\partial v}{\partial \theta} - \frac{z}{r^2}\frac{\partial^2 w}{\partial \theta^2} + \frac{1}{2r^2}\left(\frac{\partial w}{\partial \theta}\right)^2 \tag{2}$$

$$\gamma_{r\theta} = \frac{1}{r}\frac{\partial u}{\partial \theta} - \frac{2z}{r}\frac{\partial^2 w}{\partial r \partial \theta} + \frac{\partial v}{\partial r} + \frac{2z}{r^3}\frac{\partial w}{\partial \theta} - \frac{v}{r} + \frac{1}{r}\frac{\partial w}{\partial r}\frac{\partial w}{\partial \theta} \tag{3}$$

where ε_r and ε_θ are the normal strains along the r and θ directions, respectively, $\tau_{r\theta}$ is the shear strain, u, v and w represent the displacements of a point on the neutral plane along r,θ and z directions, respectively. Define the stress resultants

$$N_r = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_r dz = h\sigma_r \qquad N_\theta = \int_{-h/2}^{h/2} \sigma_\theta dz = h\sigma_\theta \qquad N_{r\theta} = \int_{-h/2}^{h/2} \tau_{r\theta} dz = h\tau_{r\theta} \tag{4}$$

$$M_r = \int_{-h/2}^{h/2} z\sigma_r dz \qquad M_\theta = \int_{-h/2}^{h/2} z\sigma_\theta dz \qquad M_{r\theta} = \int_{-h/2}^{h/2} z\tau_{r\theta} dz \tag{5}$$

$$Q_r = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{rz} dz \qquad Q_\theta = \int_{-h/2}^{h/2} \tau_{\theta z} dz \tag{6}$$

where σ_r and σ_θ are the normal strains along the r and θ directions, respectively, $\tau_{r\theta}$ is the shear stress. It may be mentioned here that h may be an arbitrary function of r. Determining the stresses from (1)-(3) using the Hooke's law and substituting in (4) and (5) yields

$$\left.\begin{aligned} M_r &= -D\left[\frac{\partial^2 w}{\partial r^2} + \frac{\vartheta}{r}\frac{\partial w}{\partial r} + \frac{\vartheta}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right] \qquad M_\theta = -D\left[\vartheta\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right] \\ \mathbf{M}_{r\theta} &= -D\left(\frac{(1-\vartheta)}{r}\right)\left[\frac{\partial}{\partial \theta}\left(\frac{\partial w}{\partial r} - \frac{w}{r}\right)\right] \end{aligned}\right\} \tag{7}$$

where, $D = \dfrac{Eh^3}{12(1-\vartheta^2)}$ $\qquad D = D_0 + \sum_{i=1}^{N} D_i(r)\delta(\theta - \theta_i)$

D is total flexural rigidity of the combination and D_0 is the flexural rigidity of the parent material the D_i is for the stiffener material. E is the modulus of elasticity and n is Poisson's ratio, and from basic hook's law relationship and stress and strain relations we get the stresses in terms of displacement as below :

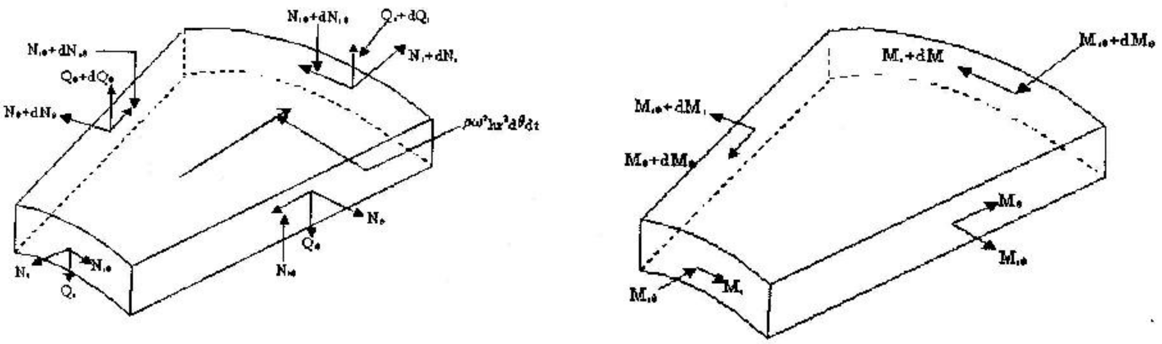


Fig. 2: Disc element with (a) forces, and (b) moments

$$\sigma_r = \frac{E}{(1-\vartheta)}\left[\frac{\partial u}{\partial r} + \frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2 + \vartheta\left(\frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{1}{2r^2}\left(\frac{\partial w}{\partial \theta}\right)^2\right)\right] \qquad \sigma_\theta = \frac{E}{(1-\vartheta)}\left[\vartheta\left(\frac{\partial u}{\partial r} + \frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2\right) + \left(\frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{1}{2r^2}\left(\frac{\partial w}{\partial \theta}\right)^2\right)\right]$$

$$\tau_{r\theta} = \frac{E}{2(1+\vartheta)}\left[\frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r} + \frac{1}{r}\left(\frac{\partial w}{\partial r}\frac{\partial w}{\partial \theta}\right)\right]$$

Using eqns (4), (5), (6), (7) and referring to fig. 2 and applying D'Alembert's principle yields the force and moment equilibrium relations as

$$\left.\begin{aligned} &\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{(\sigma_r - \sigma_\theta)}{r} + \rho r \omega^2 = 0 \\ &\frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_\theta}{\partial \theta} + \frac{2\tau_{r\theta}}{r} = 0 \end{aligned}\right\} \quad (8) \qquad \left.\begin{aligned} &\frac{\partial M_r}{\partial r} + \frac{1}{r}\frac{\partial M_{r\theta}}{\partial \theta} - \frac{M_\theta}{r} + \frac{M_r}{r} + Q_r = 0 \\ &\frac{1}{r}\frac{\partial M_\theta}{\partial \theta} + \frac{2M_{r\theta}}{r} + \frac{\partial M_{r\theta}}{\partial r} + Q_\theta = 0 \end{aligned}\right\} \quad (9)$$

$$\rho h\frac{D\omega}{Dt} = Q_r + \frac{\partial Q_r}{\partial r} + \frac{1}{r}\frac{\partial Q_\theta}{\partial \theta} + \frac{1}{r}\frac{\partial}{\partial r}\left[rh\sigma_r\frac{\partial w}{\partial r} + h\tau_{r\theta}\frac{\partial w}{\partial \theta}\right] + \frac{1}{r}\frac{\partial}{\partial \theta}\left[h\tau_{r\theta}\frac{\partial w}{\partial r} + \frac{1}{r}h\sigma_\theta\frac{\partial w}{\partial \theta}\right] \qquad (10)$$

where, ρ is the density, ω is the magnitude of the angular velocity, and D(·)/D(t) represents the material derivative. It may be noted that the in-plane and rotary inertia of the element have been neglected. The in-plane force equilibrium relations (8) are identically satisfied by assuming a scalar stress function φ such that (see [15]

$$\sigma_r = \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} - \frac{1}{2}\rho r^2\omega^2 \qquad \sigma_\theta = \frac{\partial^2 \phi}{\partial r^2} - \frac{1}{2}\rho r^2\omega^2 \qquad \tau_{r\theta} = -\left(\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial \phi}{\partial \theta}\right]\right) \qquad (11)$$

Now, solving for Q_r and Q_θ using (7) in (9) and substituting in (10), and further using (11, yields the equation of motion as below:

$$\rho h\left(\frac{\partial^2 w}{\partial t^2} + 2\omega\frac{\partial^2 w}{\partial \theta \partial t} + \omega^2\frac{\partial^2 w}{\partial \theta^2}\right) + \nabla^2\left(D\nabla^2(w)\right) - (1-\upsilon)f[w, D] - hf[w,\phi]$$

$$+\frac{1}{2}\rho\omega^2 r^2 h\nabla^2(w) + \rho\omega^2 rh\frac{\partial w}{\partial r} - \frac{\partial h}{\partial r}\left[\left(\frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} - \frac{1}{2}\rho\omega^2 r^2\right)\frac{\partial \omega}{\partial r} - \frac{1}{r}\left(\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial \phi}{\partial \theta}\right]\right)\frac{\partial w}{\partial \theta}\right] = 0$$

where,

$$f[x, y] = \frac{\partial^2 x}{\partial r^2}\left[\frac{1}{r}\frac{\partial y}{\partial r} + \frac{1}{r^2}\frac{\partial^2 y}{\partial \theta^2}\right] - 2\left(\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial x}{\partial \theta}\right]\right)\left(\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial y}{\partial \theta}\right]\right) + \frac{\partial^2 y}{\partial r^2}\left[\frac{1}{r}\frac{\partial x}{\partial r} + \frac{1}{r^2}\frac{\partial^2 x}{\partial \theta^2}\right]$$

Further the above equation is solved for non-dimensionalisation, which can be utilized for Galerkin's procedure.

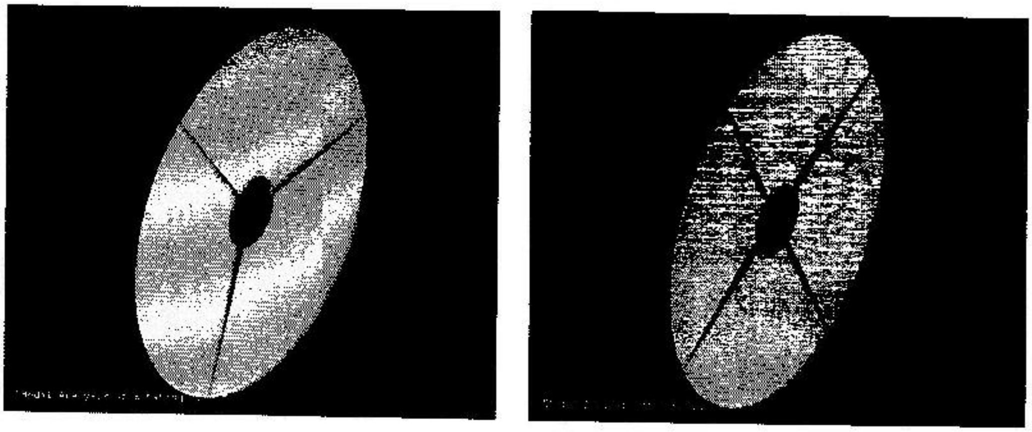
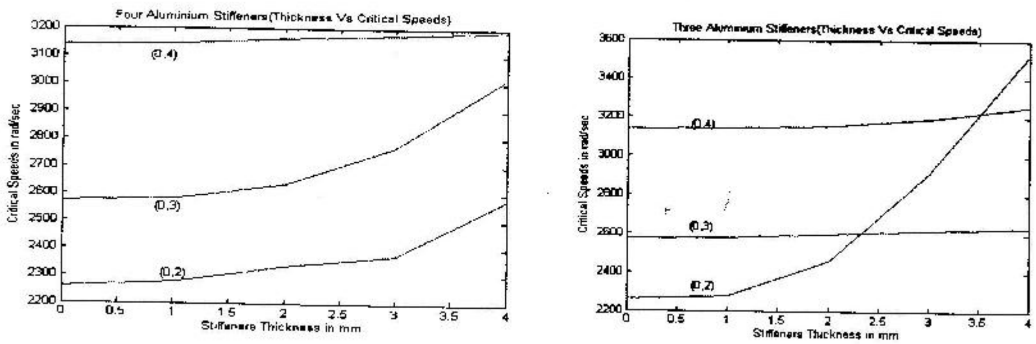


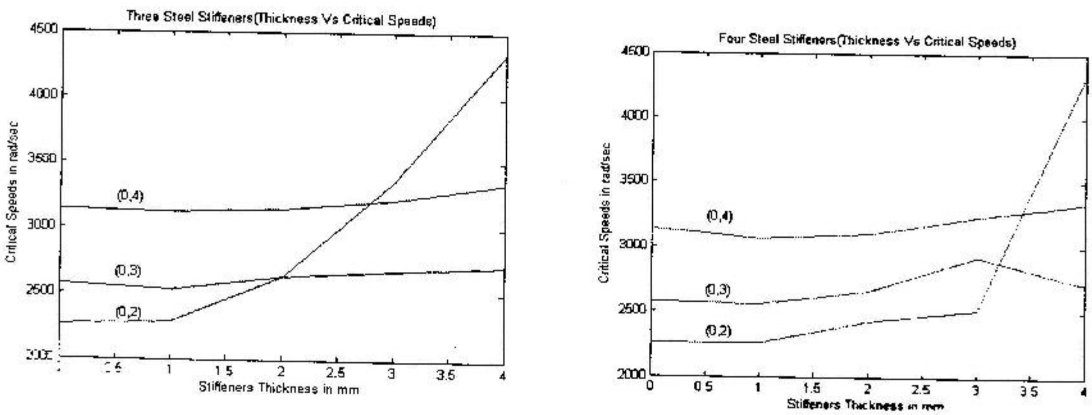
Fig. 3: Finite Element Model with (a) Three Stiffeners (b) Four Stiffeners

Results: **Effect of Stiffener Thickness (h_2) on Critical Speeds**



(a) (b)

Fig. 4: Variation critical speed with increase in thickness h_2 for a) Four b) Three, Al stiffeners.



(a) (b)

Fig. 5: Variation critical speed with increase in thickness h_2 for a) Three b) Four Steel stiffeners.

Observations:

1) The critical speeds are improves as thickness h_2 increases from 0 (no stiffeners) to 4mm in both the material combination.

2) The improvement in shown in (0,2) mode is much better than (0,3) and (0,4) modes in all cases.

3) Results are much better for h_2 = 2.5mm and above, for both material combination

4) Steel stiffeners as an edge over aluminum in improving the critical speeds.

Conclusion:

The vibration analysis of thin disc is analyzed here using finite element analysis with ANSYS 6.0 in this project work. The disc is analyzed for different material properties of stiffener and also by varying the thickness of the stiffener. It is seen that the disc becomes much stiffer as the thickness of the stiffener is increased and also the critical speeds are also increasing with the increase in stiffener thickness. The critical speeds increase with the increase in the stiffeners thickness h_2 and have better the results are better for h_2 = 3mm and above for both aluminum and steel stiffeners. Further, we find that by increasing the concentration of mass at inner radius of the disc the critical speeds are increasing.

Number of stiffeners can be decided depending the desired results to be obtained. In our work the three and four stiffeners with h_2 equal to 4mm is giving the better results.

Future Scope:

The future work on the rotating disks vibration might address on:

1) The Rotating disc with stiffeners can be studied analytically using Galerkin's procedure or any other method.

2) In all the above case we can also include the damping factor and individually studied using Finite Element Analysis.

3) Inclusion of the traction caused by friction at the interface of the shaft and the disc may lead to accurate prediction of disk natural frequencies at all rotation speeds.

4) We can also study miniature disk and spindle systems, which are generally in medical equipments like, dental drills (consist of the disk and spindle system), that run at speeds nearly 400,000rpm.

5) Environmental consideration, such as acoustics and vibration, will be more of a concern for potential users as products are refined and improved. Therefore, study of the vibroacoustics and novel damping treatments for rotating disks will become important in this regard.

References

[1] Lamb, H. and Southwell, R.V., The vibrations of a spinning disc, Proc. of Royal Society, Ser. A, Vol. 99, (1921), pp. 272-280.

[2] Nowinski, J.L., Nonlinear transverse vibrations of a spinning disc, Trans. of ASME, J. of Applied Mechanics, Vol.31, (1964), pp. 72-78.

[3] Iwan, W.D., and Moeller, T.L., The stability of a spinning elastic disk with transverse load system, Trans. of ASME, J. of Applied Mechanics, Vol. 43, (1976), pp.485-490.

[4] Benson, R.C., and Bogy, D.B., Deflection of a very flexible spinning disk due to a stationary transverse load, Trans. of ASME, J. of Applied Mechanics, Vol. 45, (1978), pp. 636-642.

[5] Shen, I.Y., and Mote, C.D., Jr., On the mechanism of instability of a circular plate under a rotating spring-mass-dashpot system, J. of Sound and Vibration, Vol. 148, No. 2, (1991), pp. 307-318.

[6] Mote, C.D., Jr., Free vibration of initially stressed circular discs, Trans. of ASME J. of Engineering for Industry, Vol. 87, (1965), pp. 258-264.

[7] Loh, H.C., and Carney III, J.F., Vibration and stability of spinning annular plates reinforced with edge beams, Trans. of ASME, J. of Applied Mechanics, Vol. 44, (1977), pp. 499-501.

[8] Renshaw, A.A., Increasing the maximum stable rotation speed of a circular disk using speed dependent clamping, J. of Sound and Vibration, Vol. 210, (1998), pp.431-439.

[9] Kim, H.R., and Renshaw, A.A., Asymmetric speed dependent tensioning of circular rotating disks, J. of Sound and Vibration, Vol. 218, (1998), pp. 65-72.

[10] Shen, I.Y., 2000, Vibration of flexible rotating disks, The Shock and Vibration Digest, Vol. 32, No. 4, (2000), pp. 267-272.

[11] Renshaw, A.A., Increasing the natural frequencies of circular disks using internal channels, J. of Sound and Vibration, Vol. 229, No. 2, (2000), pp. 355-375.

[12] Renshaw, A.A, Maximizing the natural frequencies and transverse stiffness of centrally clamped, circular disks by thickening the clamped part of the disk, Trans. of ASME, J. of Applied Mechanics, Vol. 66, (1999), pp. 1017-1021.

[13] Raman, A., and Mote, C.D., Jr., Effects of imperfection on the non-linear oscillations of circular plates spinning near critical speed, Int. J. of Non-linear Mechanics, Vol. 36, (2001), pp. 261-289.

[14] Warner, G.M., and Renshaw, A.A., Thickness profiles for rotating circular disks that maximize critical speed, Trans. of ASME, J. of Applied Mechanics, Vol. 68, (2001), pp. 505-507.

[15] Timoshenko, S., and Goodier, J.N., Theory of elasticity, McGraw-Hill Book Company,1951.

[16] Den Hartog, J.P., Advanced strength of materials, McGraw-Hill Book Company, 1952.

[17] Renshaw, A.A., and Mote, C.D., Jr., Absence of one nodal diameter critical speed modes in an axisymmetric rotating disc, Trans. of ASME, J. of Applied Mechanics,Vol. 59, (1992), pp. 687-688.