Dynamic Analysis of Rolling Bearings

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Abstract :

Analysis of rolling bearings includes finding stresses induced in the rolling element at the inner and outer raceways, which in turn involves the determination of deformations. Analysis procedures have inherent slow convergence problem, and existing gradient-based numerical schemes require good initial guesses, which is difficult to obtain and they depend upon loading conditions and bearing geometries. Often initial guesses are obtained by the trial and error method and no systematic approach has been reported in the open literature. In this paper the generalized bounds of the deformations are reported. This makes it possible to obtain deformations by the simple bisection method or any other searching method. Parametric study for cases of the positive and negative (pre-loading) clearances has been done.

1. Introduction

Much research has been done on the analysis of rolling element bearings and the advancements made were not usually of a spectacular nature but were mainly characterized by constant detailed exploration aimed at a higher degree of operational safety, higher load-carrying capacity, higher speeds and thus greater economic efficiency.

Jones (1946) developed a detailed method to calculate the distribution of loads among rolling elements of rolling bearings for the static case. Timoshenko and Goodier (1951) presented a detailed analysis on contact stresses and deformations based on the theory of elasticity. Palmgren (1959) gave a comprehensive overview on internal speeds and motions of rolling elements based on the assumption of raceway and subsequently developed equations for their calculations. Jones (1959) proposed the theoretical basis of the quasi-static analytical design. Subsequently, Jones (1960) presented an analytical method for finding the load distribution in high-speed bearings. Eschmann et al. (1985), Changsen (1991) and Harris (2000) brought out books on the theory and the design of rolling bearings in which significance was attached to the interests of practical applications like wear, heat generation, lubrication, temperature distribution etc.

In the present paper an analysis procedure for the load distribution of high-speed ball bearings have been presented. Bounds have been obtained for the faster and sure convergence of the analysis for bearings with preload and clearance. Parametric study has been done on the effects on the analysis due to various operating conditions and based on these studies general conclusions have been made.

2. Equilibrium Equations and Analysis Procedures

In high-speed bearings precise determination of rolling element contact forces at the inner and outer raceways due to the external static and centrifugal forces is important. This is done by satisfying equilibrium of forces at each rolling element (Figure 1) individually and bearing as a whole.

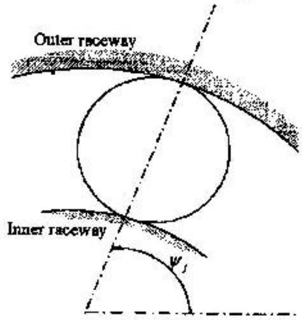

Figure1 (a) Ball at the angular position ψ_j

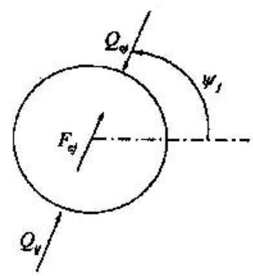

Figure1 (b) Forces acting on the ball

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Figure 1 (a) shows a typical rolling element in contact with the inner and outer raceways, Figure 1 (b) indicates fores acting on the rolling element and equilibrium equation can be written as

$$Q_{o} - Q_{i} - F_{c} = 0 \tag{1}$$

Where Q_o and Q_i are forces at the outer and inner raceways respecively, F_c is the centrifugal force and subscripts j represents a typical rolling element position. For the ball bearing, from the Hertzian contact theory we know

$$Q = K\delta^{1.5} \tag{2}$$

where Q is the force at the contact point, K is the load-deflection factor and δ is the contact deformation at the bearing raceway. On using equation (2) into equation (1), we get

$$K_0\delta_{oj}^{1.5} - K_i\delta_{ij}^{1.5} - F_{c_j} = 0 \tag{3}$$

where subscrips *o* and *i* represent *outer* and *inner* respectively, and δ_{oj} and δ_{ij} are contact deformations respectively at the outer and inner raceways at j^{th} rolling element location. The total radial deformation of the rolling element can be expressed as

$$\delta_{rj} = \delta_{ij} + \delta_{oj} \tag{4}$$

where δ_{rj} is the total radial compression at j^{th} rolling element position.

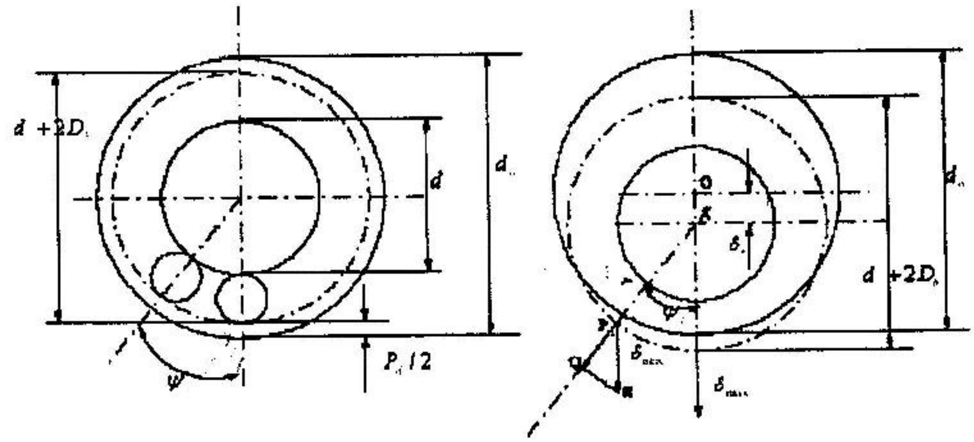


Figure 2(a) Bearing before displacement Figure 2(b) Bearing after displacement

Figure 2(a) shows a bearing before loading. Considering the geometry of the loaded bearing in Figure 2(b) the point O is the outer ring centre and A is the inner ring center, $\overline{PQ} = \delta_r \cos\psi_j$ is the total radial deflection at ψ_j and total radial compression of the rolling element at ψ_j is given by

$$\delta_{rj} = \delta_r \cos\psi_j - P_d/2 \tag{5}$$

where δ_{rj} is the total radial compression of the ball at j^{th} location, δ_r is the total radial deflection of the bearing and P_d is the diametral clearance of the bearing. From equations (4) and (5), equation (3) can be written as

$$K_0\,(\delta_r \cos\psi_j - P_d/2 - \delta_{ij})^{1.5} - K_i\,\delta_{ij}^{1.5} - F_{c_j} = 0 \tag{6}$$

The bearing equilibrium of forces at inner raceway in the direction of the applied radial load, noting equation (2), is given as

$$\frac{F_r}{K_i} - \sum_{j=i}^{j=z} \delta_{ij}^{1.5} \cos\psi_j = 0 \tag{7}$$

where Z is the number of rolling elements in the bearing. Equations (6) and (7) represent the system of simultaneous nonliner equations with δ_{ij} (j = 1, 2, 3, ..., Z) and δ_r as unknowns. After calculating δ_{ij} and δ_r, it possible to calculate rolling element forces at contact points, as

$$Q_{ij} = K_i\,\delta_{ij}^{\,1.5} \quad \text{and} \quad Q_{oj} = K_i\delta_{ij}^{\,1.5} + F_{cj} \tag{8}$$

Equations (6) and (7) are simultaneous nonlinear equations to be solved for δ_{ij} and δ_r. It has inherent slow convergence and existing numerical schemes require closer initial guesses to actual solutions. As the actual solutions are different for different loading conditions and bearing geometries it poses difficulty in finding initial guesses. No systematic approach bas been reported in the open literature. Based on the simple physical considerations the initial guess bounds are obtained and summarized in Tables 1 and 2 for positive clearance and preload, respectively. The bisection method is used to solve the rolling element (equation (6)) and inner raceway equilibrium equations (equation (7)). The solution algorithm is shown in the form of a flow chart in the Figure 3.

Table 1 Maximum and minimum initial guess limits when $P_d \geq 0$ (clearance)

Contact deformation and deflection of bearing	Minimum limit	Maximum limit
Deformation at inner ring, δ_{ij}	0	$(F_r/k_i)^{1/1.5}$
Deformation at outer ring, δ_{oj}	$(F_c/k_0)^{1/1.5}$	$((F_r+F_c)/k_0)^{1/1.5}$
Total radial compression, δ_{rj}	Minimum limit of ($\delta_{ij} + \delta_{oj}$)	Maximum limit of ($\delta_{ij} + \delta_{oj}$)
Bearing total vertical deflection, δ_r	Minimum limit of ($\delta_{rj} + ½P_d$)	Maximum limit of ($\delta_{rj} + ½P_d$)

Table 2 Maximum and minimum initial guess limits when $P_d < 0$ (preloading)

Contact deformation and deflection of bearing	Minimum limit	Maximum limit
δ_{ij}	0	$(F_r/k_i)^{1/1.5}+(\delta_i)_{pre}$
δ_{oj}	0	$\{(F_r+F_c)/k_0\}^{1/1.5}(\delta_0)_{pre}$
Total radial ball compression, δ_{rj}	Minimum limit of ($\delta_{ij} + \delta_{oj}$)	Maximum limit of ($\delta_{ij} + \delta_{oj}$)
Total radial deflection due to F_r, δ_r	0	Maximum limit of δ_{rj}

Load zones for the static (Harris, 2000) and dynamic (modified) analyses are given, respectively, as

$$\psi_L = 2\cos^{-1}(0.5P_d/\delta_r) \text{ and } \psi_L = 2\cos^{-1}\left(\{P_d + 2(F_c/k_0)^{1/1.5}\Big/2\delta_r\right) \tag{9}$$

where ψ_L is the load zone, the tearm $2\,(F_c/K_o)^{1/1.5}$ is the clearance (deformation produced by the contrifugal force at the outer raceway contact of the rolling element. The load zone is a function of clearances, speeds and loads.

3. Results and discussion

Parametric study on 6209-radial deep groove ball bearing is done by using the analysis procedure described in the previous section.

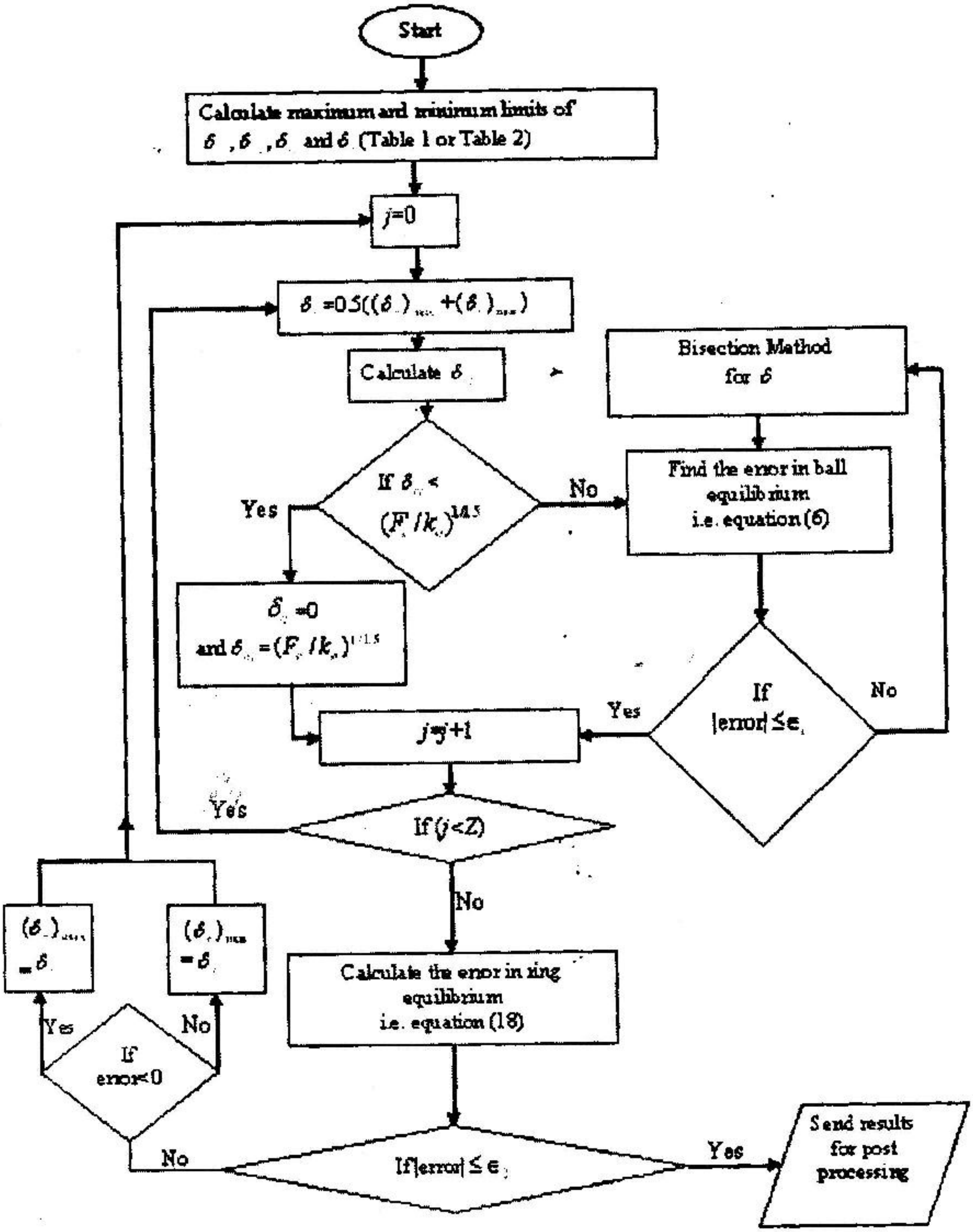


Figure 3 Flow chart of solution procedure

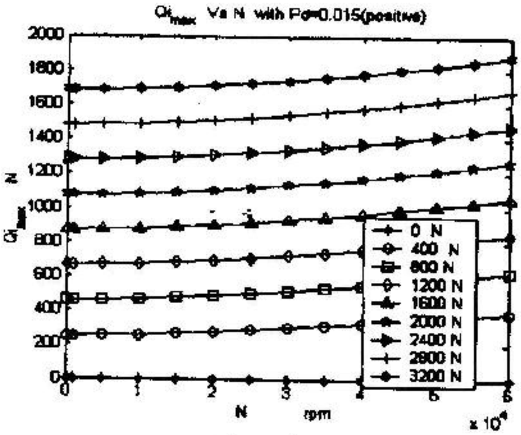
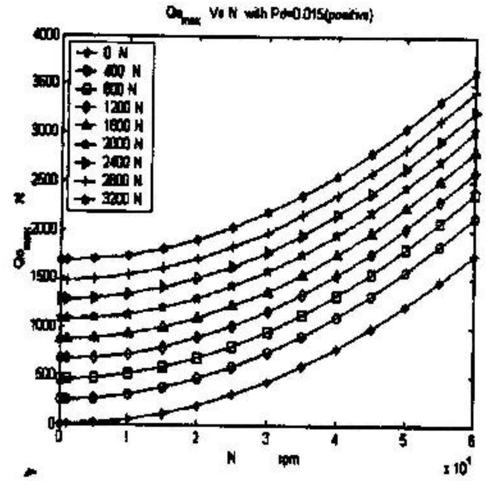


Figure 4



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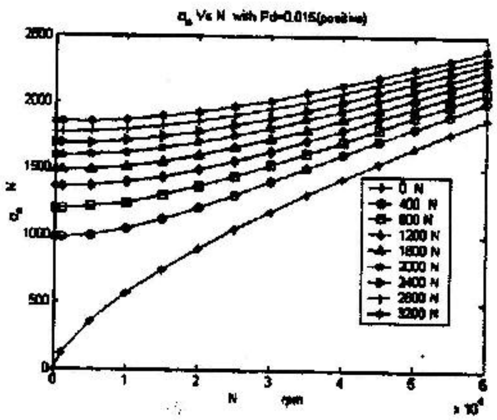

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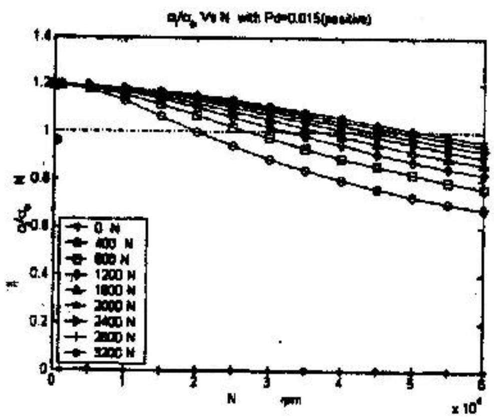

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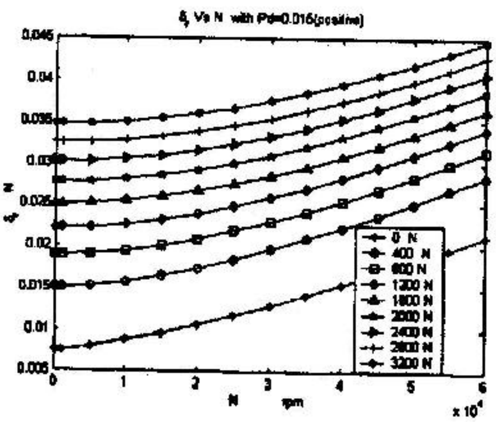

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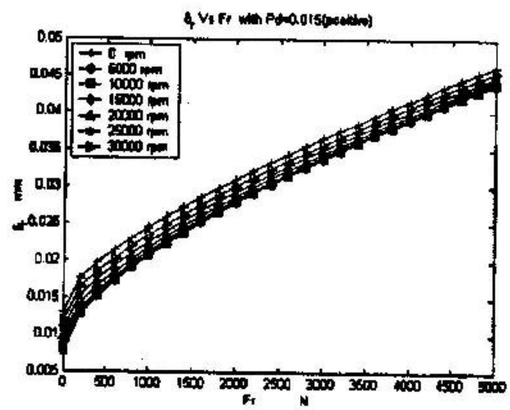
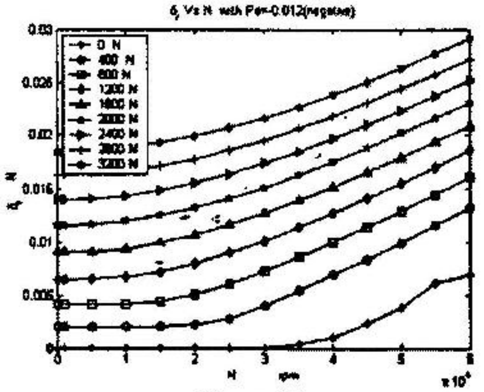


Figure 9

For negative diametral clearance



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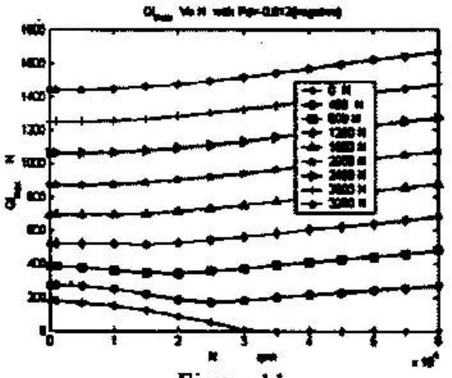

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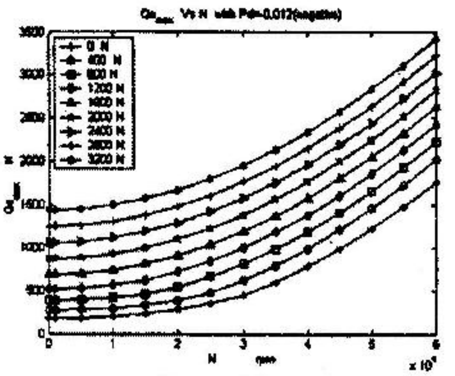

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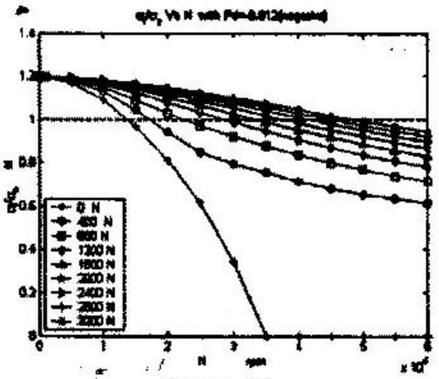

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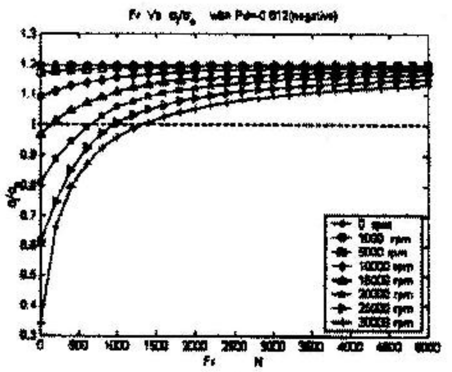

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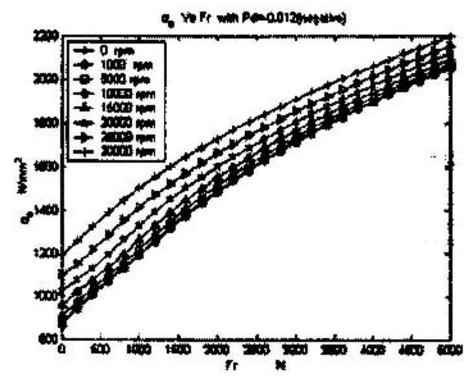

Figure 15

From Figure 4 shows the variation of maximum load at the inner raceway contact with respect to speeds and loads. With increase in the speed the maximum load increases due to the decrease in the load zone with the increase in speed. For every load there is a certain speed after which the load zone becomes so small that the entire load is carried by a single rolling element. From Figure 5 shows the variation of the maximum load at outer raceway contact versus speed for different radial load. The maximum load increases continuously with the increase in the speed as centrifugal increases with the increase in the speed. Figure 6 shows the variation of the maximum stress at outer raceway contact versus speeds for different radial loads. The maximum stress at the outer raceway increases with increase in the speed.

Figure 7 shows the variation of ratio of maximum stresses at the inner and outer raceways versus the speed for different radial loads. For the zero speed the stress at the inner raceway is more than that at the outer raceway, however, with the increase in the speed the stress at the outer raceway increases more that of the inner raceway. Thus the stress ratio is more than one at lower speeds and continuously decreases with the increase in the speed. For different loads there is a particular speed at which the ratio of the stresses equals to I. This is an important result especially for the design point of view to choose the bearing geometry such that it gives equal stresses at the inner and outer raceways. Figures 8 and 9 shows the variation of the total radial deflection versus speed and total radial deflection versus radial load, respectively. It is observed that the total deflection due the centrifugal force is very less as compared to the deflection due to the external static load. Similarly, Figures 10-15 are corresponding to the cases when the preloading is there in the bearing.

4. Conclusions

In the present paper for deep groove ball bearings load distribution, load zone and the generalized bounds of the defom1ations have been developed. This makes it possible to analyse bearings by simple bisection method or any other root searching method. These bounds are implemented in the computer code and analysis results for the cases of positive, zero and negative clearances are presented. The convergence of the results has been found to be very fast as compared to the conventional procedures.

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