Effect of Longitudinal and Transverse Roughness on Heavily Loaded EHL Elliptic Contact

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Abstract

Performance parameters of Elastohydrodynamically Lubricated (EHL) rough elliptic contact operating at heavy load ($P_H = 1.0$ GPa to 2.0 GPa) and low rolling speed (0.02 m/s to 0.5 m/s) have been computed by using an efficient numerical technique for elastic deformation. Based on the present analysis empirical relations for minimum film thickness and central film thickness have been developed and reported herein for the design of smooth and rough elliptic contacts.

1. Introduction

The current design trends (technological and economical) of Elastohydrodynamically Lubricated concentrated contacts are shifting towards increased power density, efficiency and cost reduction by subjecting the contact to higher loads, higher operating temperatures and thus thinner lubricant film thickness. Designers have to predict the minimum film thickness and optimum rolling traction for desired life of such contacts under given operating conditions. For thin lubricant film thickness, the contact behavior of elastic solids is highly sensitive to roughness topography. A small change in the distribution of the heights, widths, and curvature of the asperity peaks can have a noticeable effect on contact design. In the past, researchers [1, 2] have used stochastic modeling for studying the effect of surface roughness on average film thickness and consequently integrated contact performance parameters. Elrod [3] and Dyson [4] have presented comprehensive reviews covering the major theoretical developments in modifying the Reynolds equation to include the effect of rough surfaces, which can be characterized by a number of statistical parameters. Modeling of the contact of rough surfaces is difficult and has been treated in many papers [5-7] using a number of approaches over the past few decades. These models, especially the one given by Patir and Cheng [5], have been helpful for basic understanding of rough surface lubrication, and have provided simplified mathematical tool for handling complicated rough surfaces in lubrication field. This type of stochastic analysis, however, deals only with the global effect of surface topography. It cannot provide any detailed information about local pressure peaks and film thickness fluctuations, which are usually critical for the study of lubrication breakdown and surface failure mechanisms.

Also, there have been constant arguments as to which model is more appropriate and which parameters should be used. Since the real engineering surfaces and the lubrication phenomena are so complicated, it is probably impossible to describe their characteristics satisfactorily with a simple mathematical expression and a small number of stochastic parameters.

Recently more and more attention has been given to the analysis by incorporating deterministic model. This type of analysis uses simplified or real surface geometric profiles in the numerical solution, so that statistical parameters are no longer needed. Due to limited computational capability in the past, steady state deterministic models were developed first which have been used by Goglia et al. [8], Lubrecht et al. [9], Kweh et al. [10,11], Sadeghi [12], and Venner and Ten Napel [13]. In these models, the rough surface is stationary and the moving surface is perfectly smooth, so that the solution is completely time-independent.

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Literature survey reveals that effects of surface roughness texture and its orientation on film thickness and traction still have not been answered satisfactorily. Presently few empirical relations exist for film thickness, which are widely used to calculate EHL film thickness for smooth contact. These relations have been developed for lightly loaded (P_H<0.5 GPa) conditions due to then computational difficulty of solution of governing equations at high loads.

Rolling mills, Stone crusher machine, Regenerative air pre-heater of power plants etc generally use ball bearings, which run at extremely small speeds (1-100 rpm) under heavy loads. Researchers have not addressed performance parameter predictions of concentrated elliptic contacts in ball bearings of such machines in the past. Therefore, the objective of this paper is to evaluate the effect of longitudinal and transverse roughness on performance parameters of slow rolling heavily loaded elliptic contact by using an efficient computational technique for elastic deformation formulated by Lin and Chu [14]. Based on the present results empirical relations for minimum film thickness and central film thickness have been developed and reported herein for the design of smooth and rough concentrated contacts.

2. Governing Equations

Reynolds equation for concerned problem can be expressed as:

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{\eta}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\rho h^3}{\eta}\frac{\partial p}{\partial y}\right)=12u\frac{\partial}{\partial x}(\rho h) \tag{1}$$

The boundary conditions for Eq. (1) are: (i) pressure vanishes on the edges of the computation domain, and (ii) along the outlet boundaries $p=\partial p/\partial x=\partial p/\partial y=0$

The density and viscosity of the lubricant can be represented as:

$$\rho=\rho_0\left(1+\frac{0.58\times10^{-9}p}{1+1.68\times10^{-9}p}\right) \tag{2}$$

$$\eta=\eta_0 e^{\left[\{\ln(\eta_o)+9.37\}\{-1.0+(1.0+5.1x10^{-9}p)^z\}\right]} \tag{3}$$

The film thickness geometry can be expressed as:

$$h(x,y)=h_c+\frac{x^2}{2R_x}+\frac{y^2}{2R_y}+d(x,y)-d(0,0)+\delta(x,y) \tag{4}$$

The elastic deformation d (x, y) in Eq. (4) is evaluated by the following equation.

$$d(x,y)=\frac{2}{\pi E'}\int_{y=-\infty}^{y=+\infty}\int_{x=-\infty}^{x=+\infty}\frac{p(x,y)}{[(x-\bar{x})^2+(y-\bar{y})^2]^{1/2}}d\bar{x}d\bar{y} \tag{5}$$

The pressure $P(x, y)$ over the small rectangular element is expressed by a bi-quadratic polynomial as:

$$p=\sum_{i=0}^{2}\sum_{j=0}^{2}A_{ij}\bar{x}^i\bar{y}^j \tag{6}$$

The coefficients, A_{ij} , appearing in Eq. (6) are written in terms of grid pressures.

By substituting Eq. (6) into Eq. (5), the elastic deformation can be written as:

$$d(x,y)=\frac{2}{\pi E'}\sum_{i=0}^{2}\sum_{j=0}^{2}A_{ij}I_{ij} \tag{7}$$

Where,

$$I_{ij} = \int_{-K_4}^{K_3} \int_{-K_2}^{K_1} \frac{\bar{x}^i \bar{y}^j d\bar{x} d\bar{y}}{[(x-\bar{x})^2 + (y-\bar{y})^2]^{1/2}} \tag{8}$$

The roughness height is given as:

(i) transverse roughness:

$$\bar{\delta}(X,Y) = A_a \cos\left(\frac{\pi}{2}\frac{X}{R_a}\right) \tag{9}$$

(ii) longitudinal roughness:

$$\bar{\delta}(X,Y) = A_a \cos\left(\frac{\pi}{2}\frac{Y}{R_a}\right) \tag{10}$$

Grid pressures have been used for computation of load carrying capacity of the contact as follows:

$$w = \int_{x=-\infty}^{x=+\infty} \int_{y=-\infty}^{y=+\infty} p \, dydx \tag{11}$$

For the solution of governing equations, the computational scheme described in Ref. [14] have been adopted.

3. Results and Discussion

In the present analysis, attention is focused on the computation of minimum and central film thicknesses at high load and low rolling speeds. The input data used for numerical computation are given in Table 1. The dimensionless isothermal minimum thickness and central film thickness at various rolling/sliding speeds ($U=2.32\text{x}10^{-12}$ to $2.78\text{x}10^{-11}$), loads ($W=6.66\text{x}10^{-7}$ to $8.32\text{x}10^{-5}$), material parameter ($G=3500$ to 4500), ellipticity ratio ($k=3$ to 6), and surface roughness ($A_a= 0.2$ to 0.4; $R_a=0.25$ to 1.00) are provided in Tables 2 and 3. The relationships between load and minimum/central film thickness appear to be quite different than relations established by previous researchers under various conditions. Hamrock and Dowson [15] have observed based on their numerical results that minimum and central film thicknesses are proportional to $W^{-0.073}$ and $W^{-0.067}$ respectively. However, present study finds the corresponding exponents of loads as - 0.798 and - 0.58 respectively. This indicates that at heavy load, minimum and central film thicknesses drastically reduce. Secondly, the relationship between the material parameter (G) and H_{min} and H_c follow power law and the power exponents of G are 3.1738 and 3.0738 for minimum and central film thicknesses respectively. The trend of present results differs from those of Hamrock and Dowson formula [15] under heavy loading of contact situation.

Table 3 shows the influence of longitudinal and transverse roughness on the EHL of elliptic contacts. The effect of surface roughness on the minimum film thickness is considerably large for both transverse and longitudinal roughness patterns. However, the influence of transverse roughness on the minimum film thickness is smaller than for the longitudinal roughness cases. For low R_a, the longitudinal roughness pattern has a negative effect on the average film thickness, while for the transverse roughness case; the average film thickness is enhanced. It is observed that the empirical relations, given for minimum and central film thicknesses by [15], overestimate film thicknesses as compared with the present results. It should be mentioned here that analysis in [15] is based on smooth lightly loaded contacts, whereas the present analysis is carried out at heavy loads for rough elliptic contact by using an efficient and accurate elastic deformation methodology [14].

Based on the results presented in Tables 2 and 3 regression analyses are performed to obtain empirical relations for minimum film thickness and central film thickness in terms of operating parameters. These relations are as follows.

Minimum film thickness formula :

$$H_{min}=3.8\text{x}10^{-14}\ R_F\ U^{0.70}\ G^{3.1738}\ k^{0.5261}\ W^{-0.798}$$

where, R_F $= 1.0,$ for smooth surface

R_F $= \left(1.0-e^{-2.4R_{aL}}\right),$ for longitudinal rough surface having $A_a = 0.2$ to 0.4

R_F $= \left(1.0-0.45e^{-2.4R_{aT}}\right)$ for transverse rough surface having $A_a = 0.2$ to 0.4

Central film thickness formula:

$$H_c=50\text{x}10^{-14}\ R_F\ U^{0.66}\ G^{3.0738}\ k^{0.50}\ W^{-0.58}$$

where, R_F $= 1.0,$ for smooth surface

R_F $= \left(1.0-e^{-2.5R_{aL}}\right),$ for longitudinal rough surface having $A_a=0.2$ to 0.4

R_F $= \left(1.0-0.45e^{-2.5R_{aT}}\right)$ for transverse rough surface having $A_a=0.2$ to 0.4

These equations are valid for the following range of the operating parameters:

"W" ranges from $6.66\text{x}10^{-07}$ to $8.32\text{x}10^{-05}$, "U" ranges from $2.78\text{x}10^{-12}$ to $2.78\text{x}10^{-11}$, "G" ranges from 3500 to 4500, "k" ranges from 3 to 6, and "R_{aL}" and "R_{aT}" range from 0.25 to 1.0. It is thus believed that the present analysis will provide accurate estimation of film thickness for heavily loaded and low rolling speed elliptic contacts. It should be mentioned here that in this analysis equivalent surface asperity has been taken at stationary surface of contact for making the governing equations time independent.

4. Conclusion

The importance of this work lies in the fact that it presents for the first time empirical relations of minimum film thickness and central film thickness incorporating surface roughness for Elastohydrodynamically lubricated elliptic contacts operating under fully flooded conditions at heavy loads and low rolling speeds. These relations are expected to be very useful in design of EHL concentrated contacts.

Table 1 - Input parameters

Equivalent radius of the ball, R_x, m	0.019
Poisson's ratio of ball and race materials, ν_1 and ν_2	0.30
Young's modulus of ball and race materials, E_1, E_2, N/m^2	$2.1\text{x}10^{11}$
Density of ball and race materials,ρ, kg/m^3	7850
Pressure viscosity coefficient of lubricant, α, m^2/N	$1.82\text{x}10^{-08}$
Inlet viscosity of lubricant,η_0, Pa-s	0.096

Table 2 - Effect of load, speed, material parameter, and ellipticity on minimum film thickness and central film thickness

W	P_H (GPa)	U	G	k	H_{min} Present work	H_{min} [15]	H_c Present work	H_c [15]
6.66E-07	0.4	2.78E-12	4000	3	8.00E-06	7.20E-06	1.00E-05	9.54E-06
2.25E-06	0.6	2.78E-12	4000	3	6.00E-06	6.59E-06	7.20E-06	8.79E-06
5.33E-06	0.8	2.78E-12	4000	3	4.20E-06	6.18E-06	6.30E-06	8.30E-06
1.04E-05	1.0	2.78E-12	4000	3	1.40E-06	5.89E-06	1.60E-06	7.93E-06
1.80E-05	1.2	2.78E-12	4000	3	9.80E-07	5.66E-06	1.00E-06	7.65E-06
2.85E-05	1.4	2.78E-12	4000	3	5.00E-07	5.47E-06	6.40E-07	7.42E-06
4.26E-05	1.6	2.32E-12	4000	3	4.23E-07	4.70E-06	4.54E-07	6.40E-06
6.07E-05	1.8	2.32E-12	4000	3	3.73E-07	4.58E-06	4.24E-07	6.25E-06
8.32E-05	2.0	2.32E-12	4000	3	2.39E-07	4.47E-06	3.22E-07	6.11E-06
1.04E-05	1.0	5.57E-12	4000	3	3.20E-06	9.45E-06	3.50E-06	1.26E-05
1.04E-05	1.0	8.35E-12	4000	3	3.90E-06	1.24E-05	4.20E-06	1.66E-05
1.04E-05	1.0	1.11E-11	4000	3	4.60E-06	1.51E-05	4.80E-06	2.01E-05
1.04E-05	1.0	1.39E-11	4000	3	5.60E-06	1.76E-05	5.90E-06	2.33E-05
1.04E-05	1.0	1.67E-11	4000	3	6.30E-06	1.99E-05	6.60E-06	2.64E-05
1.04E-05	1.0	1.95E-11	4000	3	7.00E-06	2.22E-05	7.30E-06	2.93E-05
1.04E-05	1.0	2.23E-11	4000	3	8.10E-06	2.43E-05	8.40E-06	3.20E-05
1.04E-05	1.0	2.50E-11	4000	3	8.50E-06	2.62E-05	8.80E-06	3.46E-05
1.04E-05	1.0	2.78E-11	3500	3	6.50E-06	2.64E-05	6.90E-06	3.46E-05
1.04E-05	1.0	2.78E-11	3600	3	7.20E-06	2.68E-05	7.40E-06	3.51E-05
1.04E-05	1.0	2.78E-11	3700	3	7.60E-06	2.71E-05	7.80E-06	3.56E-05
1.04E-05	1.0	2.78E-11	3800	3	8.40E-06	2.75E-05	8.60E-06	3.61E-05
1.04E-05	1.0	2.78E-11	3900	3	9.00E-06	2.78E-05	9.30E-06	3.66E-05
1.04E-05	1.0	2.78E-11	4000	3	1.00E-05	2.82E-05	1.20E-05	3.71E-05
1.04E-05	1.0	2.78E-11	4100	3	1.08E-05	2.85E-05	1.10E-05	3.76E-05
1.04E-05	1.0	2.78E-11	4200	3	1.15E-05	2.89E-05	1.18E-05	3.81E-05
1.04E-05	1.0	2.78E-11	4300	3	1.21E-05	2.92E-05	1.24E-05	3.86E-05
1.04E-05	1.0	2.78E-11	4400	3	1.35E-05	2.95E-05	1.36E-05	3.90E-05
1.04E-05	1.0	2.78E-11	4500	3	1.46E-05	2.99E-05	1.49E-05	3.95E-05
1.04E-05	1.0	2.78E-11	4000	3	1.00E-05	2.82E-05	1.20E-05	3.71E-05
1.04E-05	1.0	2.78E-11	4000	4	1.10E-05	3.03E-05	1.21E-05	3.85E-05
1.04E-05	1.0	2.78E-11	4000	5	1.23E-05	3.13E-05	1.25E-05	3.92E-05
1.04E-05	1.0	2.78E-11	4000	6	1.34E-05	3.19E-05	1.36E-05	3.95E-05

Table 3 - Effect of Longitudinal and Transverse Surface Roughness on Minimum Film Thickness and Central film thickness

Roughness pattern	W	P_H (GPa)	U	G	k	A_a	R_a	H_{min} Present work	H_c Present work
Longitudinal	1.04E-05	1.0	2.78E-11	4000	3	0.2	0.25	5.07E-06	5.12E-06
	1.04E-05	1.0	2.78E-11	4000	3	0.2	0.50	6.36E-06	6.43E-06
	1.04E-05	1.0	2.78E-11	4000	3	0.2	1.00	8.44E-06	8.51E-06
	1.04E-05	1.0	2.78E-11	4000	3	0.4	0.25	4.57E-06	4.62E-06
	1.04E-05	1.0	2.78E-11	4000	3	0.4	0.50	6.87E-06	6.89E-06
	1.04E-05	1.0	2.78E-11	4000	3	0.4	1.00	8.79E-06	8.96E-06
Transverse	1.04E-05	1.0	2.78E-11	4000	3	0.2	0.25	8.12E-06	8.39E-06
	1.04E-05	1.0	2.78E-11	4000	3	0.2	0.50	8.45E-06	8.57E-06
	1.04E-05	1.0	2.78E-11	4000	3	0.2	1.00	9.34E-06	9.48E-06
	1.04E-05	1.0	2.78E-11	4000	3	0.4	0.25	7.21E-06	7.49E-06
	1.04E-05	1.0	2.78E-11	4000	3	0.4	0.50	8.52E-06	8.58E-06
	1.04E-05	1.0	2.78E-11	4000	3	0.4	1.00	9.13E-06	9.19E-06

Nomenclature

a = semi-minor length of elliptic contact area, m
a_a = amplitude of asperity,m
A_a = dimensionless amplitude of asperity, $a_a R_x/a^2$
b = semi-major length of elliptic contact area, m
$d(0,0)$ = elastic deformation at origin, m
$d(x,y)$ = elastic Deformation at a general node, m
E' = effective elastic modulus, N/m^2
E_1,E_2 = elastic modulus, N/m^2
F = friction force at surface, N
G = material parameter, $\alpha E'$
h = film thickness, m
h_c = film thickness at center of lubricated contact
h_{min} = minimum film thickness in the lubricated contact
H = non-dimensional film thickness, h/R_x
H_c = non-dimensional central film thickness, h_c/R_x
H_{min} = non-dimensional minimum film thickness, h_{min}/R_x
k = ellipticity parameter, b/a
p = pressure, Pa
P_H = maximum Hertzian pressure, $3w/(2\pi ab)$
r_a = radius of curvature of the asperity, m
R_a = dimensionless radius of curvature of asperity, r_a/a
R_{aL} = dimensionless radius of curvature of asperity in longitudinal direction
R_{aT} = dimensionless radius of curvature of asperity in transverse direction
R_F = roughness factor
R_x = radius of relative curvature of x profile, m
R_y = radius of relative curvature of y profile, m
R = equivalent radius, m
S = slip ratio, $2(u_1-u_2)/(u_1+u_2)$
t_r = traction coefficient
u_1,u_2 = tangential velocities of surfaces 1 & 2, m/s
u_r = average rolling velocity, $(u_1+u_2)/2$, m/s
U = non-dimensional speed, $u_r\eta_0/(E' R_x)$

w = total load at contact, N
W = non-dimensional load, $w/E'R_x^2$
x = coordinate, m
X = dimensionless x-coordinate
y = coordinate, m
Y = dimensionless y-coordinate
z = coefficients in Roelands' viscosity model
α = pressure-viscosity coefficient, m^2/N
$\delta(x,y)$ = shape of the asperity
η = lubricant viscosity, Pa-s
η_0 = lubricant viscosity at atmospheric conditions, Pa-s
ν_1, ν_2 = poisson's ratio
ρ = lubricant density, kg/m^3
ρ_0 = lubricant density at atmospheric condition, kg/m^3

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