Lateral Dynamic Analysis of a Typical Indian Rail-Road Vehicle

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Abstract: In this paper a numerical simulation of the lateral dynamic behavior of a railway vehicle is presented. A typical Indian railway vehicle of the AC/EMU/T (Alternating Current /Electrical Multiple Unit /Trailer) type running on broad gauge track has been chosen for the analysis. A 17 degree of freedom (dof) linear mathematical model of the vehicle has been used for the analysis. Kalker's creep theory is applied to evaluate the tangential contact forces acting between wheel and rail. Linear governing equations of motion of the vehicle on a straight track have been solved. Natural frequencies have been determined from the above mathematical model. Dynamic response studies were carried out in the frequency domain with power spectral densities of track gauge and alignment irregularities as input.

Nomenclature:

m_c, m_b, m_w	=	Mass of car body, bogie and wheel set
J_{cy}, J_{by}	=	Pitch moment of inertia of car body and bogie
J_{cx}, J_{bx}, J_{wx}	=	Roll moment of inertia of car body, bogie and wheel set
J_{cz}, J_{bz}, J_{wz}	=	Yaw moment of inertia of car body, bogie and wheel set
k_p, k_s	=	Primary and secondary stiffness in vertical direction
k_{py}, k_{sy}	=	Primary and secondary stiffness in lateral direction
k_{wf}	=	Stiffness of wheel and track
l_c	=	Half of bogie centre pin spacing
l_b	=	Semi wheel base of bogie
l_p	=	Half of primary spring spacing (lateral)
l_s	=	Half of secondary spring spacing (lateral)
l_g	=	Semi gauge length
h_c	=	Height of car cg from secondary spring centre
h_b	=	Height of bogie cg from secondary spring centre
h_w	=	Height of wheel and axle cg from primary spring centre
z_c, z_{b1}, z_{w1}	=	Vertical displacement of car, bogie and wheel set
ϕ_c, ϕ_{b1}, ϕ_{w1}	=	Roll of car body, bogie and wheel set
ψ_c, ψ_{b1}, ψ_{w1}	=	Yaw of car body bogie and wheel set
σ	=	Wheel set roll coefficient
λ_e	=	Equivalent conicity
r_o	=	Rolling radius of wheel
v	=	Linear velocity of wheel
l_b	=	Semi wheel base of bogie

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Introduction: Modern railroad vehicles are required to carry increasing loads per axle without compromising on safety, while trains consist of a larger number of vehicles than in the past. Thus the increase in complexity and sophistication of railroad vehicles has demanded a complete understanding of the dynamic behavior of the vehicle and each of its components. The study of vehicle dynamics is a difficult task. On tangent track at lower speeds of operation, rock-and-roll problems occur. At higher speeds, a vehicle may hunt or bounce severely. While negotiating a curved track, wheels may climb the rail, excessive lateral forces may occur and rails may roll over.

Brief Literature Review: Garg and Dukkipati [1] have proposed linear models for dynamic stability and curving performance. Gangadharan [2] carried out analytical and experimental studies on the dynamics of railroad vehicles with more emphasis on ride comfort. Kalker [3] developed a linear theory of rolling contact, which is being extensively used for most of the linear wheel rail contact models. Nagurka [4], in examining curving behavior of rail vehicles, included single point and two point wheel-rail contact conditions in rail vehicle equations of motion. Creep force and its moments in the lateral direction were derived and reported by Office of Research and Experiments (ORE) C116/RP 4/E [5], Garg and Dukkipati [1] and Senthil Kumar [6]. Dukkipati and Narayanaswamy [7] considered the effects of suspensions and conicity to evaluate the trade-off between dynamic stability and curving performance.

Model and Equations of Motion: A 17 dof rigid body model has been used to study the lateral dynamic behavior of an AC/EMU/T (Alternating Current / Electrical Multiple Unit / Trailer) coach used in suburban mass rapid transport in Indian Railways. For a lateral dynamics model one needs to consider lateral translation (y axis) and rotations about x and z axes (roll ϕ and yaw ψ). Fig.1 shows a rigid body model of a railroad vehicle with all the possible degrees of freedom. Lateral translation and rotation about x and z axes were the dof considered for the car body and two bogies. The four wheel and axle sets were assigned two dof each (lateral translation along y axis and rotation about z axis i.e., yaw ψ), accounting for a total of 17 dof. Equations of motion for car body, bogies and wheel and axle sets are given in equations (1) to (4). In these equations, damping of primary and secondary suspensions, as well as the effect of gyroscopic forces and moments, have been neglected.

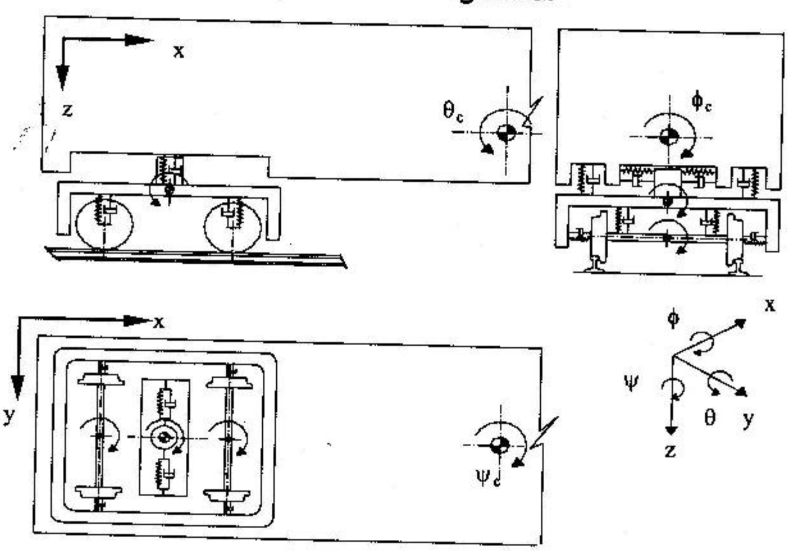

Fig .1 Rigid body model of a railroad vehicle

Car body

$$m_c\ddot{y}_c + 2k_{sy}(y_c - l_c\psi_c - h_c\phi_c - y_{b1} - h_b\phi_{b1}) + 2k_{sy}(y_c + l_c\psi_c - h_c\phi_c - y_{b2} - h_b\phi_{b2}) = 0$$
$$J_{cx}\ddot{\phi}_c - 2k_{sy}(y_c - l_c\psi_c - h_c\phi_c - y_{b1} - h_b\phi_{b1})h_c - 2k_{sy}(y_c + l_c\psi_c - h_c\phi_c - y_{b2} - h_b\phi_{b2})h_c = 0$$
$$J_{cz}\ddot{\psi}_c - 2k_{sy}(y_c - l_c\psi_c - h_c\phi_c - y_{b1} - h_b\phi_{b1})l_c + 2k_{sy}(y_c + l_c\psi_c - h_c\phi_c - y_{b2} - h_b\phi_{b2})l_c = 0 \quad (1)$$

Bogie I

$$m_b\ddot{y}_{b1}+2k_{sy}(y_{b1}+h_b\phi_{b1}-y_c+l_c\psi_c+h_c\phi_c)+2k_{py}(y_{b1}-l_b\psi_{b1}-h_w\phi_{b1}-y_{w1})$$
$$+2k_{py}(y_{b1}+l_b\psi_{b1}-h_w\phi_{b1}-y_{w2})=0$$
$$J_{bx}\ddot{\phi}_{b1}+2k_{sy}(y_{b1}+h_b\phi_{b1}-y_c+l_c\psi_c+h_c\phi_c)h_b-2k_{py}(y_{b1}-l_b\psi_{b1}-h_w\phi_{b1}-y_{w1})h_w$$
$$-2k_{py}(y_{b1}+l_b\psi_{b1}-h_w\phi_{b1}-y_{w2})h_w=0$$
$$J_{bz}\ddot{\psi}_{b1}-2k_{py}(y_{b1}-l_b\psi_{b1}-h_w\phi_{b1}-y_{w1})l_b+2k_{py}(y_{b1}+l_b\psi_{b1}-h_w\phi_{b1}-y_{w2})l_b=0 \quad (2)$$

Bogie II

$$m_b\ddot{y}_{b2}+2k_{sy}(y_{b2}+h_b\phi_{b2}-y_c-l_c\psi_c+h_c\phi_c)+2k_{py}(y_{b2}-l_b\psi_{b2}-h_w\phi_{b2}-y_{w3})$$
$$+2k_{py}(y_{b2}+l_b\psi_{b2}-h_w\phi_{b2}-y_{w4})=0$$
$$J_{bx}\ddot{\phi}_{b2}+2k_{sy}(y_{b2}+h_b\phi_{b2}-y_c-l_c\psi_c+h_c\phi_c)h_b-2k_{py}(y_{b2}-l_b\psi_{b2}-h_w\phi_{b2}-y_{w3})h_w$$
$$-2k_{py}(y_{b2}+l_b\psi_{b2}-h_w\phi_{b2}-y_{w4})h_w=0$$
$$J_{bz}\ddot{\psi}_{b2}-2k_{py}(y_{b2}-l_b\psi_{b2}-h_w\phi_{b2}-y_{w3})l_b+2k_{py}(y_{b2}+l_b\psi_{b2}-h_w\phi_{b2}-y_{w4})l_b=0 \quad (3)$$

Wheel and axle

$$m_w\ddot{y}_{w1}+2k_{py}(y_{w1}-y_{b1}+l_b\psi_{b1}+h_w\phi_{b1})-F_{y1}=0$$
$$m_w\ddot{y}_{w2}+2k_{py}(y_{w2}-y_{b1}-l_b\psi_{b1}+h_w\phi_{b1})-F_{y2}=0$$
$$m_w\ddot{y}_{w3}+2k_{py}(y_{w3}-y_{b2}+l_b\psi_{b2}+h_w\phi_{b2})-F_{y3}=0$$
$$m_w\ddot{y}_{w4}+2k_{py}(y_{w4}-y_{b2}-l_b\psi_{b1}+h_w\phi_{b2})-F_{y4}=0$$
$$J_{wz}\ddot{\psi}_{w1}+2k_{py}\psi_{w1}l_g^2-M_{z1}=0$$
$$J_{wz}\ddot{\psi}_{w2}+2k_{py}\psi_{w2}l_g^2-M_{z2}=0$$
$$J_{wz}\ddot{\psi}_{w3}+2k_{py}\psi_{w3}l_g^2-M_{z3}=0$$
$$J_{wz}\ddot{\psi}_{w4}+2k_{py}\psi_{w4}l_g^2-M_{z4}=0 \quad (4)$$

The phenomenon of creep appears when two rigid bodies are pressed against each other with force and allowed to roll over each other. The creep forces develop because of the difference in strain rates of the two bodies in the contact region. Kalker [3] developed a linear theory of rolling contact, which has been used here. The creep forces and moments are given by

$$F_{yi}=-2f_{22}\left[\frac{\dot{y}_{wi}}{v}-\psi_{wi}\right]-2f_{23}\left[\frac{\dot{\psi}_{wi}}{v}+\frac{\sigma y_{wi}}{r_o l_b}\right] \quad (5)$$

$$M_{zi}=-2f_{11}l_b\left[\frac{l_b\psi_{wi}}{v}+\frac{y_{wi}\lambda_e}{r_o}\right] \quad (6)$$

where i = 1, 2, 3, 4 refer to the four wheel sets. f_{11}, f_{22} and f_{23} are Kalker's creep coefficients with subscripts 1, 2 and 3 representing longitudinal, lateral and normal directions respectively. The equation of motion of a vehicle system can be written in the form

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{0\} \quad (7)$$

Dynamic Response Calculations: Determination of the eigenvalues is the first step towards understanding the dynamic behaviour of any system. Natural frequencies of the above lateral dynamics model are shown in Table 1.

Table 1 Natural frequencies of vehicle (Hz)

1	0.82	5	1.26	9	5.93	13	127.50
2	0.82	6	3.56	10	9.07	14	127.50
3	0.82	7	4.49	11	9.14	15	127.50
4	0.82	8	5.93	12	127.50		

Subsequently lateral displacement and acceleration have been computed for the lateral dynamic inputs to the railway vehicle. Lateral profile or alignment and gauge are the two main irregularities of the track considered for lateral dynamic studies. These irregularities are random in nature and are therefore described by their power spectral densities (PSDs) in the frequency domain. For the present study, the data on Indian railway tracks were obtained from the work of Iyengar and Jaiswal [8]. Figs. 2 and 3 show track irregularity PSDs for alignment and gauge for a vehicle speed of 30 kmph. Considering random track irregularity PSDs for alignment and gauge as inputs, equation (7) becomes

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{F(t)\} \tag{8}$$

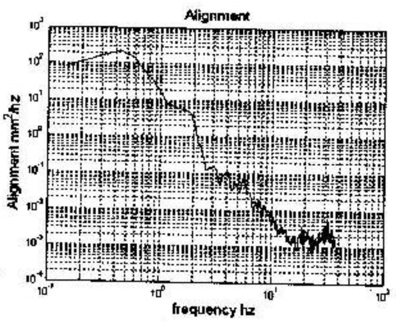

Fig. 2 Track alignment irregularity

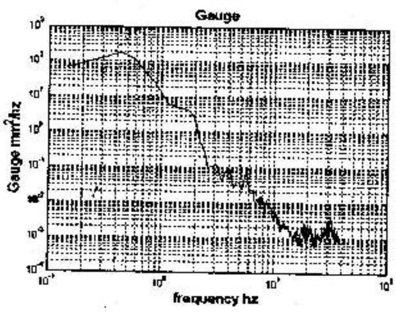

Fig. 3 Track gauge irregularity

The 17 dof model can be treated as a system with eight random disturbances due to track irregularities at each of the eight rail wheel contact points. If the input from the left rail is considered to be completely correlated with that of the right rail, then the system can be simplified to a case of four random loadings p(t), q(t), r(t) and s(t) at the wheel rail contact points. It is further assumed that the input is space correlated between the successive wheels on each rail; hence q(t), r(t) and s(t) reproduce p(t) after time lags of τ_1, τ_2 and τ_3 corresponding to wheel base b_1 (2.896 m), b_2 (14.63 m) and b_3 (17.526 m) as shown in Fig. 4. Therefore $S_p(f) = S_q(f) = S_r(f) = Ss(f)$ = PSD of p(t), q(t), r(t) and s(t) respectively. From the theory of random vibration, $S_x(f)$, the PSD of response x(t) can be calculated knowing the receptances and PSDs of the inputs from equation (8).

$$|S_x(f)| = \Big\{ |\alpha_{xp}|^2 + |\alpha_{xq}|^2 + |\alpha_{xr}|^2 + |\alpha_{xs}|^2 + 2|\alpha_{xp}||\alpha_{xq}|\cos\phi_1 + 2|\alpha_{xr}||\alpha_{xp}|\cos\phi_2 + 2|\alpha_{xs}||\alpha_{xp}|\cos\phi_3 + 2|\alpha_{xq}||\alpha_{xr}|\cos\phi_4 + 2|\alpha_{xq}||\alpha_{xs}|\cos\phi_5 + 2|\alpha_{xr}||\alpha_{xs}|\cos\phi_6 \Big\} S_p(f) \tag{9}$$

Here α_{xp}, α_{xq}, α_{xr} and α_{xs} are the receptances for harmonic excitation. Phase angles ϕ can be written in terms of the wheel bases and vehicle velocity as

$$\phi_1 = 2\pi f b_1/v,\ \phi_2 = 2\pi f b_2/v,\ \phi_3 = 2\pi f b_3/v, \phi_4 = 2\pi f b_4/v,\ \phi_5 = 2\pi f b_5/v,\ \phi_6 = 2\pi f b_6/v \tag{10}$$

Figs. 5 and 6 show the lateral acceleration and displacement response PSDs for alignment input and Figs. 7 and 8 those for gauge input at the centre of gravity (cg) of the car for a vehicle speed of 30 kmph. The variations of root mean square acceleration and displacement responses at car cg for alignment and gauge inputs with respect to speed are presented in Fig. 9 and Fig. 10.

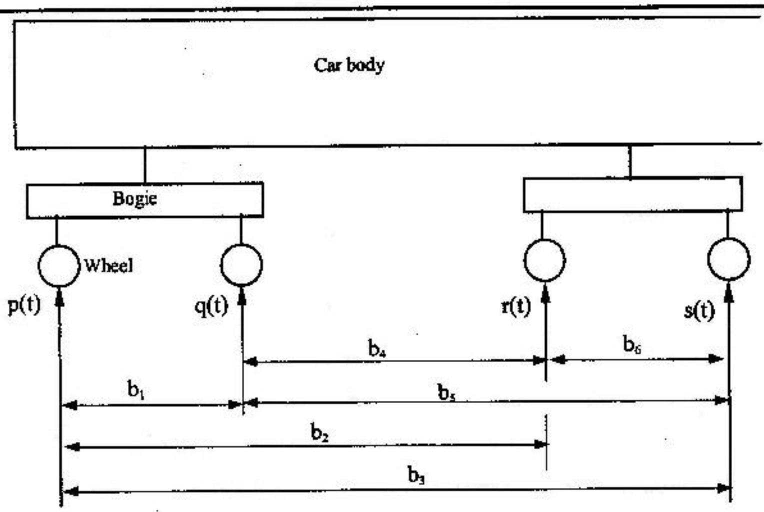

Fig. 4 Points of application of random load

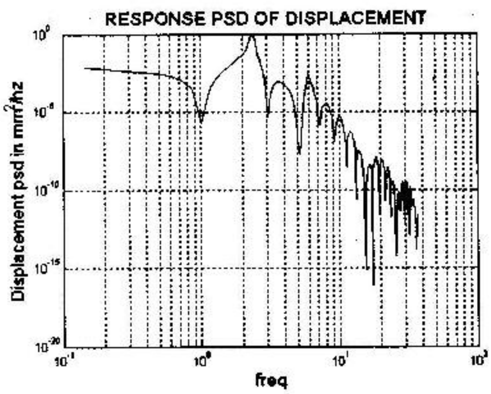

Fig. 5 Displacement response for alignment

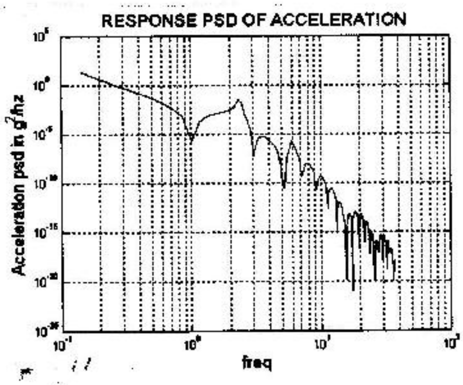

Fig. 6 Acceleration response for alignment

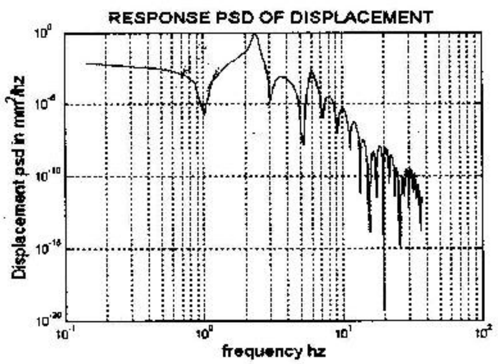

Fig. 7 Displacement response for gauge

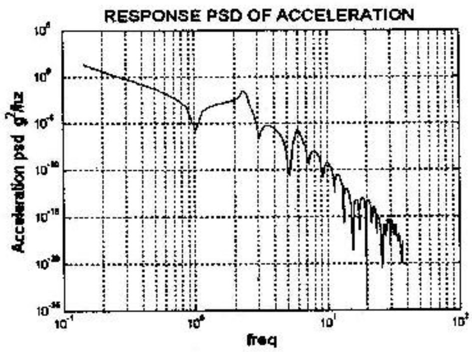

Fig. 8 Acceleration response for gauge

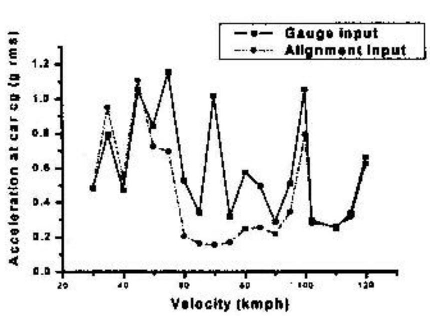

Fig. 9 Acceleration at car cg (g rms) versus velocity (kmph)

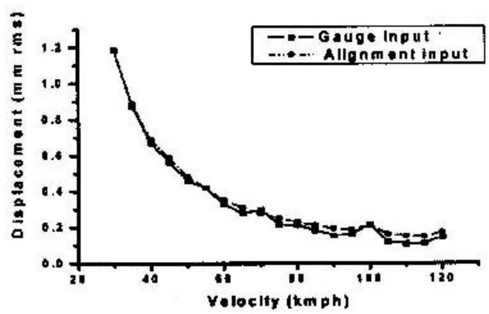

Fig. 10 Displacement at car cg (mm rms) versus velocity (kmph)

Summary and Conclusions: Lateral dynamic analysis has been carried out for a typical Indian railway vehicle of the AC/EMU/T (Alternating Current /Electrical Multiple Unit /Trailer) type running on broad gauge track. The lateral dynamic natural frequencies have been computed using a 17 dof model. Using random vibration theory, the lateral displacement and acceleration response at car cg have been calculated in the frequency domain at different speeds with PSDs of the random track alignment and gauge irregularities given as input. The root mean square values of lateral displacements and accelerations were found to be speed dependent as is to be expected, since the input PSDs (in g^2/Hz vs. Hz) themselves are speed dependent.

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Appendix

AC/EMU/T vehicle details	
Vehicle particulars	Weight in tonnes
Tare weight	31
Gross weight	58
Axle capacity	13
Unsprung mass - two wheels and axle	1.6
Sprung mass with bogie bolster	2.4
Total weight of one bogie	5.6
Tare weight of car body	19.8
Salient dimensions of an AC/EMU/T coach, in mm	
Overall length of the car body	20726
Width of the car body	3658
Overall height of the car body from rail level	3810
Height of the coach floor from rail level	1047
Distance between bogie centers	14630
Rigid wheel base of bogies	2896
Wheel diameter on tread	952
Gauge	1676

Spring particulars	Suspension properties primary	secondary	
	mm	Outer (mm)	Inner (mm)
Wire diameter	33.5	40	30
Inner diameter	175	218	148
Mean diameter	208.5	258	178
Free height	304	281	281
Home height	200	160	165
No. of effective coils	4.25	3	4.5
Deflection /1000kg	17.99	19.74	30.73
Combined deflection		12.01	
Residual deflection in %	31.73	25	
Deflection free to tare	28	31	
Deflection tare to gross	31	41	
Deflection gross to shock	12	15	
Deflection tare to shock	43	56	
Deflection ratio Py : Sy	43:57		
Damping particulars			
Primary vertical damping	2000 N at 10 cm/s (20 Ns/mm)		
Secondary vertical damping	6000 N at 10 cm/s (60 Ns/mm)		

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